On the Use of Mismatched Wiener Filtering for the Characterization of Non-Stationary Channels

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***Abstract*—We develop a method for the determination of local regions in time in which a channel can be approximated as stationary. Contrary to previous results in literature relying on to some extent arbitrary measures and thresholds, we consider a realistic (flat fading) channel estimator and relate the size of local quasi-stationarity regions to the degradation of the mean squared channel estimation error due to mismatched statistics. As the evaluation of the mean squared error turns out to be difficult, we give an approximate expression. Using channel measurements, we exemplarily evaluate the local quasi-stationarity regions based on the actual and the approximate mean squared error, and we find that the results show strong similarities.**

I. Introduction

An important simplification of the statistical modeling of linear wireless channels is the assumption of first and second order stationarity. These wide-sense stationary (WSS) and uncorrelated scattering (US) channels are called WSSUS channels [1]. This assumption results in mathematical simplifications, but it also has a physical justification, as large scale effects such as shadow fading change the statistics of the channel only slowly in comparison to the coherence time. In [1], this leads to the quasi-WSSUS model where the channel is divided into WSSUS regions. A framework for the treatment of non-stationary channels, which fulfill the doubly underspread assumption, is presented in [2].

From the analysis of measured wireless channels [3], [4], we know that a realistic approach is the assumption of local quasi-stationarity (LQS) regions, i.e., the approximation of the channel as a stationary process inside non-stationary regions of a certain size. An important open problem is the determination of the size of these regions. Even when restricting to a comparison of time-varying power spectral densities (PSDs), various measures can be defined. The usual approach of selecting, to some extent arbitrary, measures and to compare them to arbitrary thresholds is far from being satisfactory. We overcome this problem by relating the non-stationarity characterization of the channel to an algorithmic view.

Contribution: We describe a method for the determination of LQS regions in time based on the performance degradation of a realistic algorithm. Our concept is based on a comparison of power spectral densities (PSDs) as in [5]; however, we extend the approach to a filter using pilot symbols to estimate a time-varying frequency-flat fading channel from noisy observations. The concepts are applied to non-stationary doubly underspread channels. The performance degradation is described by the degradation of the mean squared error (MSE) of a finite-length Wiener filter due to mismatched statistical knowledge. The performance degradation the system engineer is willing to accept is reflected by a threshold, which determines the size of the LQS regions. As the MSE evaluation turns out to be difficult, we also give an approximate expression based on infinite-length Wiener filtering. Finally, we provide an exemplary analysis using channel measurements; we observe that the actual and the approximate evaluation of the MSE result in the same trends in the LQS regions.

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II. System Model

In the complex baseband, the matched-filtered, symbol-sampled received signal is given by[1]

$$y[m] = h[m]x[m] + n[m] \tag{1}$$

where the additive noise $n[m]$ is a white jointly proper Gaussian process with known variance $\sigma_n^2 > 0$ and the time-varying channel transfer function $h[m]$ is a jointly proper zero-mean random process independent of $n[m]$. The transmitted sequence $x[m]$ consists of data symbols and periodically inserted pilot symbols with period L at positions $m = nL$ for $n \in \mathbb{Z}$. Without loss of generality, we assume real- and positive-valued pilot symbols with magnitude σ_p.

We introduce the normalized Doppler $\nu = \nu' T$ with the Doppler ν' and the symbol duration T. We assume that channel sampling by pilot symbols fulfills the Nyquist criterion

$$L < \frac{1}{2\nu_{\max}} \tag{2}$$

with the maximal (normalized) Doppler frequency $\nu_{\max} = \nu'_{\max} T$. The assumption of a maximum Doppler frequency is realistic as any movement, be it from the transmitter, the receiver, or the scatterers in the environment occurs with a finite velocity. Therefore, the channel transfer function $h[m]$ is bandlimited and a sufficient statistic of $h[m]$ is obtained by regularly sampling $h[m]$ with period L.

[1] Strictly speaking, (1) is not a sufficient statistic; however, it is an approximation for a (Doppler-)dispersion underspread channel.

III. Mismatched Wiener Filtering

We now describe *mismatched* Wiener filtering for WSS channels. Here mismatch refers to the use of wrong statistical knowledge of the channel, but not of the noise. The mismatched statistics of the channel correspond to a bandlimited process fulfilling (2). The considered Wiener filter uses noisy observations at pilot positions for the estimation of the channel process. In the matched case the Wiener filter is a linear minimum MSE (LMMSE) estimator.

A. Finite-Length Filtering

We first consider the finite-length filtering case with N_p pilot symbols and a pilot spacing L. Denoting the filtering length as N, we have $N_d = N - N_p$ data symbols. Without loss of generality, we assume that the interval starts with a pilot symbol. The temporally ordered vectors $\mathbf{y}_p$, $\mathbf{h}_p$, and $\mathbf{n}_p$ are the noisy observations, the channel, and the noise at pilot positions, respectively. We thus obtain

$$\mathbf{y}_p = \mathbf{h}_p \sigma_p + \mathbf{n}_p. \tag{3}$$

The mismatched MSE matrix follows as [6]

$$\begin{aligned}\tilde{\mathbf{R}}_e = \mathbf{R}_h &+ \tilde{\mathbf{R}}_{h;h_p} \left(\tilde{\mathbf{R}}_{h_p} + \mathbf{I}_{N_p}\sigma_n^2/\sigma_p^2\right)^{-1} \\ &\times \left(\mathbf{R}_{h_p} + \mathbf{I}_{N_p}\sigma_n^2/\sigma_p^2\right)\left(\tilde{\mathbf{R}}_{h_p} + \mathbf{I}_{N_p}\sigma_n^2/\sigma_p^2\right)^{-1} \tilde{\mathbf{R}}_{h;h_p}^H \\ &- \tilde{\mathbf{R}}_{h;h_p}\left(\tilde{\mathbf{R}}_{h_p} + \mathbf{I}_{N_p}\sigma_n^2/\sigma_p^2\right)^{-1} \mathbf{R}_{h;h_p}^H \\ &- \mathbf{R}_{h;h_p}\left(\tilde{\mathbf{R}}_{h_p} + \mathbf{I}_{N_p}\sigma_n^2/\sigma_p^2\right)^{-1} \tilde{\mathbf{R}}_{h;h_p}^H \end{aligned} \tag{4}$$

where $\mathbf{R}_h = \mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\}$ is the autocorrelation matrix of the channel at all positions, $\mathbf{R}_{h_p} = \mathrm{E}\left\{\mathbf{h}_p\mathbf{h}_p^H\right\}$ is the autocorrelation matrix of the channel at pilot positions, and $\mathbf{R}_{h;h_p} = \mathrm{E}\left\{\mathbf{h}\mathbf{h}_p^H\right\}$ is the cross-correlation matrix between the channel at all positions and the channel at pilot positions only. $\tilde{\mathbf{R}}_h$ and $\tilde{\mathbf{R}}_{h;h_p}$ are the corresponding mismatched correlation matrices assumed by the estimator. The average mismatched MSE over all positions is

$$\tilde{\sigma}_{e,N,L}^2 = \frac{1}{N}\operatorname{tr}\left\{\tilde{\mathbf{R}}_e\right\}. \tag{5}$$

Using the real-valued and non-negative PSD of the channel $C_h\left(e^{j2\pi\nu}\right)$, we obtain

$$[\mathbf{R}_h]_{k,l} = \int_{-\frac{1}{2}}^{\frac{1}{2}} C_h\left(e^{j2\pi\nu}\right) e^{j2\pi(k-l)\nu} d\nu \tag{6}$$

with $[\mathbf{R}_{h_p}]_{k,l} = [\mathbf{R}_h]_{(k-1)L+1,(l-1)L+1}$ and $[\mathbf{R}_{h;h_p}]_{k,l} = [\mathbf{R}_h]_{k,(l-1)L+1}$. The mismatched correlation matrices are obtained analogously with the real-valued and non-negative mismatched PSD of the channel $\tilde{C}_h\left(e^{j2\pi\nu}\right)$.

B. Infinite-Length Filtering

In the following, we consider the infinite-length Wiener filter using observations at pilot positions only. The (noisy) observations at pilot positions are

$$y[nL] = h[nL]\sigma_p + n[nL] \tag{7}$$

and estimation is performed on the l-th position relative to the pilot grid with $l = 0, ..., L-1$. It can be shown that the PSD of the mismatched error process is [6]

$$\begin{aligned}\tilde{C}_e\left(e^{j2\pi\nu}\right) &= C_h\left(e^{j2\pi\nu}\right) + \left(C_h\left(e^{j2\pi\nu}\right) + \frac{L\sigma_n^2}{\sigma_p^2}\right) \\ &\times \frac{\tilde{C}_h^2\left(e^{j2\pi\nu}\right)}{\left(\tilde{C}_h\left(e^{j2\pi\nu}\right) + \frac{L\sigma_n^2}{\sigma_p^2}\right)^2} - \frac{2C_h\left(e^{j2\pi\nu}\right)\tilde{C}_h\left(e^{j2\pi\nu}\right)}{\tilde{C}_h\left(e^{j2\pi\nu}\right) + \frac{L\sigma_n^2}{\sigma_p^2}}.\end{aligned} \tag{8}$$

The (position-independent) mismatched MSE is obtained as

$$\tilde{\sigma}_{e,\infty,L}^2 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \tilde{C}_e\left(e^{j2\pi\nu}\right) d\nu. \tag{9}$$

Note that the MSE of the finite-length filtering case $\tilde{\sigma}_{e,N,L}^2$ converges to the infinite one $\tilde{\sigma}_{e,\infty,L}^2$ for $N \to \infty$ and assuming that $N = N_p L$ holds [6].

IV. Application to Non-Stationary Channels

So far we have assumed a WSS channel. In a real scenario, the wireless channel is non-stationary; however, the channel can be assumed to be stationary inside small regions [1], [2]. A mismatched Wiener filtering in these assumed stationarity regions is valid, and thus with the corresponding MSE, we have a reasonable way to relate the performance of a realistic channel estimation algorithm to specific channel properties. However, the resulting MSE, i.e., (5) with (4), does not have a convenient form for a simple evaluation, e.g., matrix inversions are required. In contrast, the infinite-length filtering results in an expression for the MSE, i.e., (9) with (8), that allows for a simplified evaluation. The infinite-length filtering approach is strictly speaking not appropriate; however, since the coefficients of the Wiener filter decay over time and, as we will see in Section IV-A, common wireless channels have an effectively finite correlation, it can be considered as meaningful. Now we show how to adapt the results of Section III to the estimation of non-stationary channels.

In [2], the class of doubly-underspread (DU) channels is introduced for time- and frequency-varying channels. In the special case of only time-varying (frequency-flat fading) channels as considered here, these channels are *dispersion* underspread with a maximal Doppler $\nu_{\text{max}} \ll 1$ and *correlation* underspread with a maximal correlation in Doppler $\Delta\nu_{\text{max}} \ll \nu_{\text{max}}$. This essentially means that the stationarity in time $N_s = \frac{1}{\Delta\nu_{\text{max}}}$ is much larger than the coherence in time $N_c = \frac{1}{\nu_{\text{max}}}$, which itself is much larger than 1. The DU assumption is usually fulfilled for wireless channels, see [3] for an example in an urban macrocell scenario.

A. Channels with an Effectively Finite Correlation

Due to the correlation underspread property, i.e., $N_c \ll N_s$, we have a time-varying autocorrelation function of the channel that is approximately zero outside a finite interval, or that is *effectively* timelimited. Thus, we assume the channel to be correlated over a finite interval only[2]. We will see that

[2]Note that a *strictly* bandlimited signal cannot be *strictly* timelimited, but only *effectively* timelimited, see [7] for a detailed discussion.

the estimator of the statistics of the channel presented in Section V performs a windowing over the channel process; thus, it is also based on the assumption of a finite correlation of the channel. The finite-length filtering approach of length N makes only use of the correlation properties of the channel for time differences $-(N-1),...,N-1$, see Section III-A. Thus, it only uses the channel correlation on an interval of length $N' = 2N-1$, i.e., a maximum of $N-1$ time instants in each time direction. We choose N' to be equal to the assumed finite correlation length of the channel, and assume N' to be a multiple of L. Note that we need to choose $N_c \ll N' \leq N_s$ for our assumption to be valid. In order to simplify the exposition, we restrict to a WSS channel for the remainder of Section IV-A. This leads to

$$C_h\left(e^{j2\pi\nu}\right) = \sum_{\Delta m=-(N-1)}^{N-1} R_h[\Delta m]\; e^{-j2\pi\Delta m\nu}. \qquad (10)$$

Due to the finite correlation assumption, we can substitute $R_h[\Delta m]$ by the inverse discrete Fourier transform (DFT) of samples of $C_h\left(e^{j2\pi\nu}\right)$; thus, we obtain

$$\begin{aligned} & C_h\left(e^{j2\pi\nu}\right) \\ &= \frac{1}{N'}\sum_{k'=-(N-1)}^{N-1} C_h\left(e^{j2\pi\frac{k'}{N'}}\right)\sum_{\Delta m=-(N-1)}^{N-1} e^{j2\pi\Delta m\left(\frac{k'}{N'}-\nu\right)}. \end{aligned} \qquad (11)$$

1) Finite-Length Filtering: In the finite-length filtering case of length N, we can insert (11) into (6) to obtain the correlation matrices using samples of the matched and mismatched PSD of the channel as

$$\begin{aligned} [\mathbf{R}_h]_{k,l} &= \frac{1}{N'}\sum_{k'=-(N-1)}^{N-1} C_h\left(e^{j2\pi\frac{k'}{N'}}\right)e^{j2\pi(k-l)\frac{k'}{N'}} \\ \left[\mathbf{R}_{h_p}\right]_{k,l} &= \frac{1}{N'}\sum_{k'=-(N-1)}^{N-1} C_h\left(e^{j2\pi\frac{k'}{N'}}\right)e^{j2\pi(k-l)L\frac{k'}{N'}} \\ \left[\mathbf{R}_{h;h_p}\right]_{k,l} &= \frac{1}{N'}\sum_{k'=-(N-1)}^{N-1} C_h\left(e^{j2\pi\frac{k'}{N'}}\right)e^{j2\pi((k-1)-(l-1)L)\frac{k'}{N'}} \end{aligned}$$

and accordingly for the mismatched case. In matrix notation, we obtain

$$\begin{aligned} \mathbf{R}_h &= \breve{\mathbf{F}}_h^H\breve{\mathbf{C}}_h\breve{\mathbf{F}}_h \\ \mathbf{R}_{h_p} &= \breve{\mathbf{F}}_{h,L}^H\breve{\mathbf{C}}_h\breve{\mathbf{F}}_{h,L}; \qquad \tilde{\mathbf{R}}_{h_p} = \breve{\mathbf{F}}_{h,L}^H\breve{\tilde{\mathbf{C}}}_h\breve{\mathbf{F}}_{h,L} \\ \mathbf{R}_{h;h_p} &= \breve{\mathbf{F}}_h^H\breve{\mathbf{C}}_h\breve{\mathbf{F}}_{h,L}; \qquad \tilde{\mathbf{R}}_{h;h_p} = \breve{\mathbf{F}}_h^H\breve{\tilde{\mathbf{C}}}_h\breve{\mathbf{F}}_{h,L}. \end{aligned} \qquad (12)$$

The $N'\times N$ matrix $\breve{\mathbf{F}}_h$ and the $N'\times N_p$ matrix $\breve{\mathbf{F}}_{h,L}$ are sub-matrices of the $N'\times N'$ DFT matrix $\breve{\mathbf{F}}$:

$$\begin{aligned} [\breve{\mathbf{F}}_h]_{k,l} &= [\breve{\mathbf{F}}]_{k,l} = 1/\sqrt{N'}\exp\left(-j2\pi(k-1)(l-1)/N'\right) \\ [\breve{\mathbf{F}}_{h,L}]_{k,l} &= 1/\sqrt{N'}\exp\left(-j2\pi(k-1)(l-1)L/N'\right) \end{aligned} \qquad (13)$$

i.e., $\breve{\mathbf{F}}_h$ contains the first N columns of $\breve{\mathbf{F}}$, and $\breve{\mathbf{F}}_{h,L}$ contains only the columns of $\breve{\mathbf{F}}_h$ coinciding with pilot positions. The $N'\times N'$ matrices $\breve{\mathbf{C}}_h$ and $\breve{\tilde{\mathbf{C}}}_h$ are diagonal matrices containing regular samples of the matched and mismatched PSD of the channel, respectively:

$$\left[\breve{\mathbf{C}}_h\right]_{k,k} = \begin{cases} C_h\left(e^{j2\pi\frac{k-1}{N'}}\right), & \text{for } 1\leq k\leq \frac{N'+1}{2} \\ C_h\left(e^{j2\pi\frac{k-1-N'}{N'}}\right), & \text{for } \frac{N'+1}{2} < k \leq N' \end{cases}$$

and accordingly for the mismatched case[3].

2) Infinite-Length Filtering: In the infinite-length filtering case, we can insert (11) and its mismatched equivalent into (8) to obtain the PSD of the error process. As this still necessitates an integration over the PSD of the error process to obtain the MSE in (9), we perform an approximation often made in the literature, where an integration over a continuous variable is replaced by a summation over (available) samples of that variable. One should, however, be careful under what conditions such an approximation is reasonable. Rewriting (10), we obtain

$$\begin{aligned} C_h\left(e^{j2\pi\nu}\right) &= \sum_{\Delta m=-\infty}^{\infty} R_h[\Delta m]\,\mathrm{rect}_{N-1}\left[\Delta m\right]e^{-j2\pi\Delta m\nu} \\ &= C_h\left(e^{j2\pi\nu}\right)\circledast\frac{\sin(\pi\nu N')}{\sin(\pi\nu)},\; -\frac{1}{2}\leq\nu<\frac{1}{2} \end{aligned} \qquad (14)$$

with $\mathrm{rect}_{N-1}[m] = 1, \forall|m|\leq N-1$ and 0 else, and with $\circledast$ denoting a circular convolution. We call a PSD that fulfills (14) a smooth PSD, as such a PSD changes slowly over ν due the circular convolution with the smoothing function. From (8), we can deduce that the maximal Doppler of $\tilde{C}_e\left(e^{j2\pi\nu}\right)$ is equal to the maximum of the maximal Dopplers of $C_h\left(e^{j2\pi\nu}\right)$ and $\tilde{C}_h\left(e^{j2\pi\nu}\right)$; this implies that the coherence times are the same as well, and thus we can conclude that a smooth PSD of the channel results in a rather smooth PSD of the error. Defining

$$\breve{\tilde{C}}_e\left(e^{j2\pi\nu}\right) = \tilde{C}_e\left(e^{j2\pi\nu}\right)\circledast\frac{\sin(\pi\nu N')}{\sin(\pi\nu)},\; -\frac{1}{2}\leq\nu<\frac{1}{2} \qquad (15)$$

we get a representation based on the sampled PSD as in (11)

$$\breve{\tilde{C}}_e\left(e^{j2\pi\nu}\right) = \frac{1}{N'}\sum_{k=-(N-1)}^{N-1}\breve{\tilde{C}}_e\left(e^{j2\pi\frac{k}{N'}}\right)\frac{\sin\left(\pi\left(\nu N'-k\right)\right)}{\sin\left(\pi\frac{\nu N'-k}{N'}\right)}.$$

Approximating the error process as finitely correlated on an interval of length N', i.e., $\tilde{C}_e\left(e^{j2\pi\nu}\right)\approx\breve{\tilde{C}}_e\left(e^{j2\pi\nu}\right)$, the required samples of $\breve{\tilde{C}}_e\left(e^{j2\pi\nu}\right)$ are obtained from the mismatched error PSD $\tilde{C}_e\left(e^{j2\pi\nu}\right)$ in (8) and accordingly for the matched case. In [6], we show that for rectangular PSDs of the channel process with $\frac{1}{2L}-\frac{1}{2N'}<\nu_{\max}<\frac{1}{2L}$, this rather heuristic approximation leads to bounds on the MSEs obtained by finite-length filtering.

[3]Note that our decomposition of the correlation matrices in (12) is different from the singular value decomposition used in, e.g., the mismatched (frequency correlation-based) estimator in [8].

B. Time-Dependent Power Spectral Density

The local scattering function (LSF) is an extension of the scattering function for WSSUS channels to the non-stationary case [2]. In the case of a frequency-flat fading channel, it is a time-dependent PSD in the Doppler domain. We adapt the approach in [2] to discrete-time channels:

$$C_h\left(m; e^{j2\pi\nu}\right) = \sum_{\Delta m=-\infty}^{\infty} R_h[m;\Delta m] e^{-j2\pi\Delta m\nu} \tag{16}$$

where $R_h[m;\Delta m] = \mathrm{E}\left\{h[m]h^*[m-\Delta m]\right\}$ is the correlation function of the channel. Note that a WSS channel has a constant LSF over time and is uncorrelated in Doppler.

The LSF has some deficiencies with respect to the PSD of a stationary process, e.g., it is not guaranteed to be real-valued and non-negative. For DU channels, it is possible to define generalized LSFs (GLSFs) which are smoothed versions of the LSF and do not have the above deficiencies [2]. In the discrete-time and frequency-flat fading case, we define the GLSF as

$$\begin{aligned} C_h^{(\Phi)}\left(m; e^{j2\pi\nu}\right) = &\sum_{m'=-\infty}^{\infty} \int_{-\frac{1}{2}}^{\frac{1}{2}} C_h\left(m'; e^{j2\pi\nu'}\right) \\ &\times \Phi\left(m-m'; e^{j2\pi(\nu-\nu')}\right) d\nu' \end{aligned} \tag{17}$$

with

$$\Phi\left(m; e^{j2\pi\nu}\right) = \sum_{k=1}^{K} \gamma_k \sum_{\Delta m=-\infty}^{\infty} g_k^*[-m] g_k[-m-\Delta m] e^{-j2\pi\Delta m\nu}$$

where $g_k[m]$ are windowing functions normalized to unit-energy, K is their number, and $\gamma_k \geq 0$ fulfills $\sum_{k=1}^{K}\gamma_k = 1$. GLSFs of DU channels are real-valued, non-negative, and approximately equivalent to the LSF [2].

C. Mean Squared Error

We now describe the evaluation of the MSE based on the time-dependent and sampled PSD $C_h^{(\Phi)}[m;k] = C_h^{(\Phi)}(m; e^{j2\pi k/N'})$.

1) Finite-Length Filtering: For the finite-length filtering case, we obtain the mismatched MSE at time instant m using statistical knowledge at time instant m' as

$$\tilde{\sigma}^2_{e,N,L}[m,m'] = \frac{1}{N}\,\mathrm{tr}\left\{\tilde{\mathbf{R}}_e[m,m']\right\} \tag{18}$$

with the mismatched MSE matrix adapted from (4) with (12)

$$\begin{aligned} \tilde{\mathbf{R}}_e[m,m'] = &\breve{\mathbf{F}}_h^H \breve{\mathbf{C}}_h \breve{\mathbf{F}}_h + \breve{\mathbf{F}}_h^H \breve{\tilde{\mathbf{C}}}_h \breve{\mathbf{F}}_{h,L} \\ &\left(\breve{\mathbf{F}}_{h,L}^H \breve{\tilde{\mathbf{C}}}_h \breve{\mathbf{F}}_{h,L} + \mathbf{I}_{N_p}\sigma_n^2/\sigma_p^2\right)^{-1} \left(\breve{\mathbf{F}}_{h,L}^H \breve{\mathbf{C}}_h \breve{\mathbf{F}}_{h,L} + \mathbf{I}_{N_p}\sigma_n^2/\sigma_p^2\right) \\ &\times \left(\breve{\mathbf{F}}_{h,L}^H \breve{\tilde{\mathbf{C}}}_h \breve{\mathbf{F}}_{h,L} + \mathbf{I}_{N_p}\sigma_n^2/\sigma_p^2\right)^{-1} \breve{\mathbf{F}}_{h,L}^H \breve{\tilde{\mathbf{C}}}_h^H \breve{\mathbf{F}}_h \\ &- \breve{\mathbf{F}}_h^H \breve{\tilde{\mathbf{C}}}_h \breve{\mathbf{F}}_{h,L} \left(\breve{\mathbf{F}}_{h,L}^H \breve{\tilde{\mathbf{C}}}_h \breve{\mathbf{F}}_{h,L} + \mathbf{I}_{N_p}\sigma_n^2/\sigma_p^2\right)^{-1} \breve{\mathbf{F}}_{h,L}^H \breve{\mathbf{C}}_h^H \breve{\mathbf{F}}_h \\ &- \breve{\mathbf{F}}_h^H \breve{\mathbf{C}}_h \breve{\mathbf{F}}_{h,L} \left(\breve{\mathbf{F}}_{h,L}^H \breve{\tilde{\mathbf{C}}}_h \breve{\mathbf{F}}_{h,L} + \mathbf{I}_{N_p}\sigma_n^2/\sigma_p^2\right)^{-1} \breve{\mathbf{F}}_{h,L}^H \breve{\tilde{\mathbf{C}}}_h^H \breve{\mathbf{F}}_h. \end{aligned}$$

The matched MSE at time instant m is obtained as

$$\sigma^2_{e,N,L}[m] = \frac{1}{N}\,\mathrm{tr}\left\{\mathbf{R}_e[m]\right\} \tag{19}$$

where the matched MSE matrix

$$\begin{aligned} \mathbf{R}_e[m] = &\breve{\mathbf{F}}_h^H \breve{\mathbf{C}}_h \breve{\mathbf{F}}_h - \breve{\mathbf{F}}_h^H \breve{\mathbf{C}}_h \breve{\mathbf{F}}_{h,L} \\ &\times \left(\breve{\mathbf{F}}_{h,L}^H \breve{\mathbf{C}}_h \breve{\mathbf{F}}_{h,L} + \mathbf{I}_{N_p}\sigma_n^2/\sigma_p^2\right)^{-1} \breve{\mathbf{F}}_{h,L}^H \breve{\mathbf{C}}_h^H \breve{\mathbf{F}}_h \end{aligned}$$

follows from this mismatched one with $\breve{\tilde{\mathbf{C}}}_h = \breve{\mathbf{C}}_h$. The difference to Section IV-A1 consists in the use of the (diagonal) time-dependent PSD matrices with

$$\left[\breve{\mathbf{C}}_h\right]_{k,k} = \begin{cases} C_h^{(\Phi)}[m;k-1], & \text{for } 1 \leq k \leq \frac{N'+1}{2} \\ C_h^{(\Phi)}[m;k-1-N'], & \text{for } \frac{N'+1}{2} < k \leq N' \end{cases}$$

$$\left[\breve{\tilde{\mathbf{C}}}_h\right]_{k,k} = \begin{cases} C_h^{(\Phi)}[m';k-1], & \text{for } 1 \leq k \leq \frac{N'+1}{2} \\ C_h^{(\Phi)}[m';k-1-N'], & \text{for } \frac{N'+1}{2} < k \leq N' \end{cases}.$$

2) Infinite-Length Filtering: For the infinite-length filtering case, we obtain the mismatched MSE based on (9) and the finite correlation approximation of the error process in Section IV-A2. This leads to the replacement of the integration in (9) by a summation. Thus, with (8), the mismatched MSE at time instant m using statistical knowledge at time instant m' simplifies to

$$\begin{aligned} \tilde{\sigma}^2_{e,\overline{\infty},L}[m,m'] &= \frac{1}{N'} \sum_{k=-(N-1)}^{N-1} \tilde{C}_e\left(m,m'; e^{j2\pi\frac{k}{N'}}\right) \\ &= \frac{1}{N'} \sum_{k=-(N-1)}^{N-1} \left(C_h^{(\Phi)}[m;k] + \left(C_h^{(\Phi)}[m;k] + \frac{L\sigma_n^2}{\sigma_p^2}\right) \right. \\ &\left. \times \frac{{C_h^{(\Phi)}}^2[m';k]}{\left(C_h^{(\Phi)}[m';k] + \frac{L\sigma_n^2}{\sigma_p^2}\right)^2} - \frac{2C_h^{(\Phi)}[m;k]\,C_h^{(\Phi)}[m';k]}{C_h^{(\Phi)}[m';k] + \frac{L\sigma_n^2}{\sigma_p^2}} \right) \end{aligned} \tag{20}$$

where the index $\overline{\infty}$ denotes the approximate evaluation and $\tilde{C}_e\left(m,m'; e^{j2\pi\nu}\right)$ is the (time-dependent) error PSD at time m using statistical knowledge of the channel at time m'. The matched MSE at time instant m simplifies to

$$\sigma^2_{e,\overline{\infty},L}[m] = \frac{1}{N'} \sum_{k=-(N-1)}^{N-1} \frac{C_h^{(\Phi)}[m;k]}{\frac{\sigma_p^2}{L\sigma_n^2} C_h^{(\Phi)}[m;k] + 1}. \tag{21}$$

With $\tilde{\sigma}^2_{e,\overline{\infty},L}[m,m']$ and $\sigma^2_{e,\overline{\infty},L}[m]$, we have found approximate, but simplified, evaluations of the mismatched and matched MSE, respectively. In [6], we show that for rectangular PSDs of the channel process with $\frac{1}{2L} - \frac{1}{2N'} < \nu_{\text{max}} < \frac{1}{2L}$, $\sigma^2_{e,N,L}[m] \geq \sigma^2_{e,\overline{\infty},L}[m]$ holds.

D. Local Quasi-Stationarity

In order to relate the size of LQS regions to a performance measure, we use the MSE degradation at time m due to the use of mismatched statistics of the channel, i.e., the statistics of the channel at time m':

$$\eta_{N,L}[m,m'] = \frac{\tilde{\sigma}^2_{e,N,L}[m,m']}{\sigma^2_{e,N,L}[m]} - 1 \tag{22}$$

The finite-length filtering case is denoted by the index N and the approximate one, based on infinite-length filtering and the finite correlation approximation of the error process, by substituting N by $\overline{\infty}$. We choose a threshold η_{th} corresponding to the maximum acceptable performance loss due to mismatched statistics of the channel. This threshold thus has a meaning and its choice can be well motivated. Defining the set

$$\mathcal{M}_{N,L}[m] = \{m' \mid \eta_{N,L}[m, m'] < \eta_{\text{th}}\} \tag{23}$$

we obtain time-dependent LQS times

$$T_{\text{LQS},N,L}[m] = |\mathcal{C}_{N,L}[m]|\, T \tag{24}$$

where $\mathcal{C}_{N,L}[m]$ is the connected subset of $\mathcal{M}_{N,L}[m]$ containing m and having maximum cardinality. In [6], we show that for rectangular PSDs of the channel process with $\frac{1}{2L} - \frac{1}{2N'} < \nu_{\max} < \frac{1}{2L}$ and a pilot spacing $L = 1$, the approximate MSE degradation is equal to the MSE degradation based on finite-length filtering. The same holds for the resulting LQS regions.

V. Analysis of a Measured Channel

We estimate $C_h^{(\Phi)}[m; p]$ from a single measurement run by applying the estimator in [9] to the flat fading setting:

$$\hat{C}_h^{(\Phi)}[m; p] = \frac{T_{\text{meas}}}{T} \frac{1}{K} \sum_{k=0}^{K-1} \Bigg| \sum_{m'=-\lfloor N'/(2L) \rfloor}^{\lceil N'/(2L) \rceil - 1} g_k^*[m'] \times h((m + m')T_{\text{meas}}) e^{-j2\pi L \frac{pm'}{N'}} \Bigg|^2 \tag{25}$$

for $-\lfloor N'/(2L) \rfloor \leq p \leq \lceil N'/(2L) \rceil - 1$ and $\hat{C}_h^{(\Phi)}[m; p] = 0$ else. Here $h(mT_{\text{meas}})$ denotes the measured samples of the (continuous-time) channel. We choose the measurement samples to be the pilot symbols, i.e., $T_{\text{meas}} = LT$. The urban macrocell channel measurements used here are relevant for the 3GPP Long Term Evolution, see [3] for details. We consider vertically polarized propagation at 2.5 GHz with the uniform linear array at base station (BS) 1 at a height of 25 m as transmitter (TX) and the lower uniform circular array of the mobile terminal (MT) on track 9a-9b as receiver (RX). The specific link consists of an antenna at the TX and the antenna oriented to the right of the MT at the RX. We have verified the DU assumption $\Delta\nu_{\max} \ll \nu_{\max} \ll 1$; in [3], we show that it is fulfilled with $\Delta\nu_{\max} \approx \frac{0.03}{L}$ and $\nu_{\max} \approx \frac{0.31}{L}$. The windows $g_k[m]$ in (25) are chosen as discrete prolate spheroidal sequences [10] with length $\frac{N'}{L} = 29$. This results in a coherence time $T_c = N_c T \approx 0.04$ s, $N'T = 0.38$ s, and a stationarity time $T_s = N_s T \approx 0.43$ s. We thus fulfill $N_c \ll N' \leq N_s$. The number of windows is $K = 4$. We perform the analysis on a zero-mean channel, i.e., we remove the time-dependent mean, estimated using 29 time instants and a frequency bandwidth of 5 MHz, from the channel. Then we use (25) to estimate the GLSF and additionally average the estimate over a bandwidth of 5 MHz to improve the estimation.

In Fig. 1, we show exemplarily the LQS regions d_{LQS} based on the MSE degradation with (18), (19) and with (20), (21). We see that both LQS regions show strong similarities. The approximate LQS regions seem to lower-bound the LQS regions based on finite-length filtering. This is supported by $\sigma^2_{e,N,L}[m] \geq \sigma^2_{e,\overline{\infty},L}[m]$ for rectangular PSDs of the channel process with $\frac{1}{2L} - \frac{1}{2N'} < \nu_{\max} < \frac{1}{2L}$, which is shown in [6].

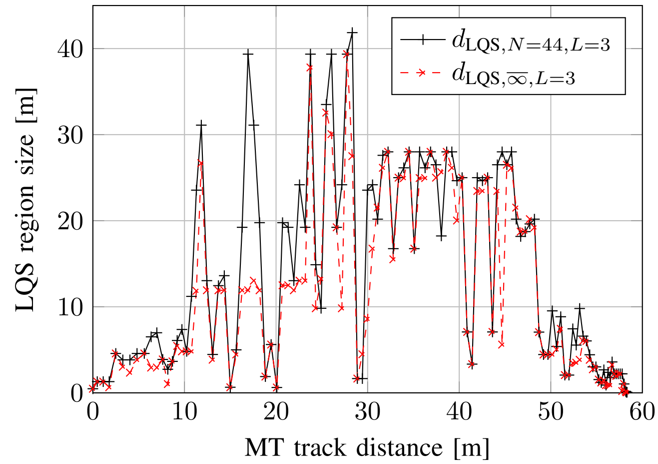

Fig. 1. LQS regions with $\eta_{\text{th}} = 0.1$, $N = 44$, $L = 3$, and the average signal-to-noise ratio $\overline{|h(mT_{\text{meas}})|^2}\sigma_p^2/\sigma_n^2 = 7$ dB

VI. Conclusion

We have developed a method to determine LQS regions in time, i.e., regions in which a channel can be approximated as stationary in time. Contrary to previous results in the literature, we relate the size of LQS regions to the performance degradation of a realistic channel estimator. The estimator is a finite-length Wiener filter estimating a time-varying frequency-flat fading channel. The performance degradation is described by the MSE degradation and the threshold is chosen according to the performance loss one is willing to accept. Furthermore, we have given an approximate expression of the MSE, which allows for a simplified evaluation. Exemplarily, we have evaluated the actual and the approximate MSE using channel measurements, and we have observed that the resulting LQS regions show strong similarities in their evolution.

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