Joint Reduction of Peak-to-Average Power Ratio and Out-of-Band Power in OFDM Systems

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***Abstract*—The high peak-to-average power ratio is a major drawback of OFDM systems. Many PAPR reduction techniques have been proposed in the literature, among them a method that uses a subset of tones that do not carry any data, but are modulated such that the PAPR of the resulting time domain signal is minimized.**

Another problem of OFDM systems is the high out-of-band power caused by the sidelobes of the modulated tones. The OBP can be reduced by modulating reserved tones at the edges of the occupied spectrum so that the sidelobes of the data carriers are reduced.

In this paper, we propose to consider both optimization problems jointly. This way, the amount of PAPR and OBP reduction can be significantly enhanced in comparison to a system that performs two separate optimization steps. Furthermore, the joint reduction algorithm offers more flexibility, because the relative weighting of the two optimization criteria can easily be changed, resulting in a smooth trade-off curve.

I. Introduction

Multicarrier techniques such as orthogonal frequency division multiplexing (OFDM) offer a high spectral efficiency and are therefore well suited for wireless transmission systems. The simple one-tap frequency domain equalizer makes OFDM especially interesting for low-cost applications such as mobile cellular networks. However, the superposition of many carriers results in a high peak-to-average power ratio (PAPR), which is a major drawback of OFDM systems due to the nonlinear behaviour of power amplifiers (PA). Either the PA is operated with a large back-off factor, or strong peaks will drive it into saturation. The resulting clipping effects give rise to inter-carrier interference and increased spectral sidelobes. A large back-off factor, however, decreases the power efficiency while increasing the cost of the devices.

Many PAPR reduction techniques have been proposed in the literature [1], among them the *tone reservation* technique [2], which will be the focus of this paper. It uses a subset of carriers, called *reserved tones* (RTs), which do not carry any data, but are modulated with complex weighting factors such that the PAPR of the resulting time domain signal is reduced. An advantage of tone reservation over other PAPR reduction techniques is that no side information must be transmitted. If channel state information is available to the transmitter, the RTs can be assigned to the weakest subcarriers, where data transmission would hardly be possible anyway. This way, the loss in data rate due to the introduction of RTs can be minimized.

Another potential drawback (depending on the scenario) of OFDM systems is the high out-of-band power (OBP) due to the sidelobes of the subcarriers. In [3], an OFDM-based overlay system was studied, which uses gaps within the spectrum assigned to another transmission system. A high OBP of the overlay system can lead to significant interference with the legacy system and therefore has to be reduced. Another scenario where a high OBP can be problematic is the uplink of a cellular network using orthogonal frequency division multiple access (OFDMA). In such a system, each user is assigned a small block of subcarriers. Since the oscillators of the mobiles are not synchronized, and due to different Doppler shifts, the signals arrive with different frequency offsets at the base station and the orthogonality between the subcarriers is lost. The resulting multiple access interference (MAI) can be reduced by minimizing the OBP of all signals.

A possible solution to this problem is again based on RTs [3]. A few subcarriers at the edges of the spectrum are weighted with complex factors such that the sidelobes of the data carriers are reduced.

In this paper, we propose to use a single set of RTs that are modulated such that both the PAPR and the OBP are reduced. We present simulation results that show the superior performance of the joint optimization compared to a system that uses two disjoint sets of RTs, one being used solely for PAPR reduction, and the other only for OBP reduction. A further advantage of the proposed algorithm is that the trade-off between the two optimization criteria can easily be adapted to the current situation. For instance, in an OFDMA system with power control in the uplink, mobiles that are close to the base station transmit with reduced average power. In this case, the PAPR does not pose a severe problem anymore, and the main emphasis of the joint optimization can be put onto OBP reduction.

The paper is structured as follows. Section II gives an overview of the OFDM system that we are considering, and introduces the notation that will be used throughout the paper. Section III first reviews the algorithms from [2] and [3], which focus on the reduction of PAPR and OBP, respectively, followed by our proposal of a joint optimization algorithm. In Section IV, simulation results are presented that show the

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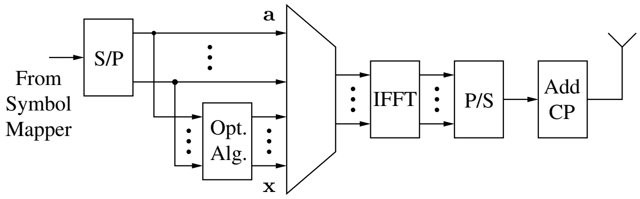


Fig. 1. Block diagram of an OFDM transmitter using the tone reservation technique

performance of the algorithms. The summary in Section V concludes the paper.

II. System Model

We consider a complex baseband OFDM transmitter as depicted in Fig. 1. The algorithm presented in this paper works on each OFDM symbol separately, without any dependencies between consecutive symbols, so we omit an OFDM symbol index for simplicity. The available subcarriers are separated into two sets: Data tones and RTs. Both transmitter and receiver know the position of the RTs. After mapping bits to data symbols, they are passed to the optimization algorithm, which calculates complex weights for the RTs such that both the PAPR and the OBP of the resulting transmitted signal are reduced. These weights are then multiplexed with the data symbols and assigned to their respective subcarriers. The complete set of symbols is transformed into the time domain via an IFFT operation, and the signal is transmitted after prepending the cyclic prefix (CP). At the receiver, the reserved tones are simply discarded.

We use the following notation: The FFT-length is denoted by N_c, and the total symbol length (including the CP) by $N_s = N_c + N_{cp}$. For simplicity, we define the normalized frequency $\nu = (f - f_c)T$, where f_c is the carrier frequency, and T is the OFDM symbol duration (without CP). The normalized center frequencies of the subcarriers are then the integers $\mathcal{K} = \{-N_c/2, -N_c/2+1, \ldots, N_c/2-1\}$. $\mathcal{M} \subset \mathcal{K}$ is the set of modulated subcarriers. The subcarriers $\mathcal{M}_d \subset \mathcal{M}$ are modulated with data symbols[1] $\mathbf{a} \in \mathcal{A}^{|\mathcal{M}_d|}$, drawn i.i.d. from a complex-valued alphabet $\mathcal{A}$. The symbols are assumed to have unit variance, i.e. $E\{\mathbf{a}\mathbf{a}^H\} = I_{|\mathcal{M}_d|}$. The subcarriers $\mathcal{M}_r = \mathcal{M} \setminus \mathcal{M}_d$ are the reserved tones which are weighted with factors $\mathbf{x} \in \mathbb{C}^{|\mathcal{M}_r|}$.

III. Reduction Algorithms

A. PAPR reduction

The discrete-time transmitted signal is generated by an IFFT of the carrier weights, where L-times oversampling is used in order to approximate the peaks in the continuous-time signal. According to [2], an oversampling factor of $L = 4$ is sufficient and has been used in our simulations for the evaluation of the PAPR. With the IFFT matrix $\tilde{Q} \in \mathbb{C}^{LN_c \times N_c}$ with entries

$$\tilde{Q}_{n,k} = \frac{1}{N_c} \exp\left(2\pi j \frac{nk}{LN_c}\right), 0 \le n \le LN_c - 1, k \in \mathcal{K}, \quad (1)$$

the transmitted signal can be written as $\tilde{\mathbf{s}} = \tilde{Q}(\tilde{\mathbf{a}} + \tilde{\mathbf{x}})$, where the IFFT input vectors $\tilde{\mathbf{a}}, \tilde{\mathbf{x}} \in \mathbb{C}^{N_c}$ contain the subcarrier weights from $\mathbf{a}$ and $\mathbf{x}$ at the frequency bins $\mathcal{M}_d$ and $\mathcal{M}_r$, respectively, and are zero otherwise. The PAPR of this signal is defined as[2]

$$PAPR = \frac{||\tilde{\mathbf{s}}||_\infty^2}{\frac{1}{LN_c}||\tilde{\mathbf{s}}||_2^2} = \frac{||Q\mathbf{x} + \mathbf{s}||_\infty^2}{\frac{1}{LN_c}||Q\mathbf{x} + \mathbf{s}||_2^2}, \quad (2)$$

where $\mathbf{s} = \tilde{Q}\tilde{\mathbf{a}}$ is the IFFT of the data carriers, and the matrix $Q \in \mathbb{C}^{LN_c \times |\mathcal{M}_r|}$ consists of the columns of $\tilde{Q}$ that correspond to the reserved tones $\mathcal{M}_r$. The task of the PAPR reduction algorithm is then to find the vector $\mathbf{x}$ that minimizes (2), given the vector of data symbols.

The complexity of the minimization problem can be reduced by assuming that the total signal energy, i.e. the denominator of (2), stays approximately constant. In this case, the problem can be formulated as

$$\text{minimize}_{\mathbf{x}} \quad ||Q\mathbf{x} + \mathbf{s}||_\infty, \quad (3)$$

where for simplicity we have taken the (strictly increasing) square root of the PAPR. Since norms are convex functions, the problem (3) can be solved by standard convex optimization algorithms [4].

B. OBP reduction

The discrete-time signal of the k-th subcarrier, modulated with $a_k = 1$ and including the cyclic prefix, is given as

$$s_k(n) = \frac{1}{N_c} \exp\left(2\pi j \frac{nk}{LN_c}\right), -LN_{cp} \le n \le LN_c - 1. \quad (4)$$

The corresponding continuous-frequency spectrum is

$$\begin{aligned} S_k(\nu) &= \frac{1}{N_c} \sum_{n=-LN_{cp}}^{LN_c-1} \exp\left(2\pi j \frac{nk}{LN_c}\right) \exp\left(-2\pi j \frac{n\nu}{LN_c}\right) \\ &= \frac{1}{N_c} \sum_{n=-LN_{cp}}^{LN_c-1} \exp\left(-2\pi j \frac{(\nu-k)n}{LN_c}\right) \\ &= \frac{LN_s}{N_c} D_{LN_s}\left(2\pi \frac{\nu-k}{LN_c}\right) \\ &\quad \cdot \exp\left(-\pi j(\nu-k)\frac{LN_c - LN_{cp} - 1}{LN_c}\right), \end{aligned} \quad (5)$$

where we have introduced the Dirichlet function (periodic sinc) $D_N(x) = \frac{\sin(N\frac{x}{2})}{N\sin(\frac{x}{2})}$. The spectrum of the transmitted signal is then given as the weighted superposition of all subcarrier spectra

$$S(\nu) = \sum_{k \in \mathcal{M}_r} x_k S_k(\nu) + \sum_{k \in \mathcal{M}_d} a_k S_k(\nu), \quad (6)$$

where a_k and x_k are the components of the vectors $\mathbf{a}$ and $\mathbf{x}$, respectively. The out-of-band power that we want to reduce

[1]The cardinality of a set $\mathcal{M}$ is denoted by $|\mathcal{M}|$.

[2]The norm $||\mathbf{x}||_\infty$ is the maximum of the absolute values of the components of $\mathbf{x}$.

can now be calculated as an integral over the power spectral density (PSD):

$$\int_{-\frac{LN_c}{2}}^{\nu_l} |S(\nu)|^2 d\nu + \int_{\nu_u}^{\frac{LN_c}{2}} |S(\nu)|^2 d\nu. \tag{7}$$

The values ν_l and ν_u denote the lower and upper edge of the occupied frequency band, respectively. However, since a numerical integration would be very complex, we instead evaluate the PSD only at discrete frequencies $\mathcal{V}$ close to the edges of the occupied spectrum. Let $\mathbf{S}_k(\mathcal{V}) = [S_k(\nu_1) \dots S_k(\nu_{|\mathcal{V}|})]^T \in \mathbb{C}^{|\mathcal{V}|}$ denote the vector of spectral components of the k-th subcarrier at the frequencies $\nu_i \in \mathcal{V}$. The OBP is then approximately proportional to

$$\begin{aligned} OBP &\propto \sum_{\nu_i \in \mathcal{V}} |S(\nu_i)|^2 \\ &= \sum_{\nu_i \in \mathcal{V}} \Big| \sum_{k \in \mathcal{M}_r} x_k S_k(\nu_i) + \sum_{k \in \mathcal{M}_d} a_k S_k(\nu_i) \Big|^2 \\ &= \Big|\Big| \sum_{k \in \mathcal{M}_r} x_k \mathbf{S}_k(\mathcal{V}) + \sum_{k \in \mathcal{M}_d} a_k \mathbf{S}_k(\mathcal{V}) \Big|\Big|_2^2. \end{aligned} \tag{8}$$

If we collect the column vectors $\mathbf{S}_k(\mathcal{V})$, $k \in \mathcal{M}_r$, in the matrix A, and define the vector $\mathbf{b} = \sum_{k \in \mathcal{M}_d} a_k \mathbf{S}_k(\mathcal{V})$, we can express the OBP reduction problem as

$$\text{minimize}_{\mathbf{x}} \quad ||A\mathbf{x} + \mathbf{b}||_2, \tag{9}$$

where we have again taken the square root for the sake of simplicity.

C. Joint reduction

With (3) and (9), we have derived two optimization problems for the reduction of the peak-to-average power ratio and the out-of-band power, respectively. In order to achieve a joint reduction of both values, we combine both criteria and obtain the vector-valued objective function

$$f_0(\mathbf{x}) = \begin{pmatrix} ||Q\mathbf{x} + \mathbf{s}||_\infty \\ ||A\mathbf{x} + \mathbf{b}||_2 \end{pmatrix}. \tag{10}$$

In general, there does not exist an optimal value $\mathbf{x}^*$ for the multicriterion problem $\text{minimize}_{\mathbf{x}} f_0(\mathbf{x})$ (i.e. a vector $\mathbf{x}^*$ that minimizes both the PAPR and the OBP). We therefore scalarize the problem by multiplying it with the weighting vector $\begin{pmatrix}1-\lambda, & \mu\lambda\end{pmatrix}$. The trade-off parameter $\lambda \in [0;1]$ determines the relative weighting of the two optimization criteria. By varying λ, we obtain the set of Pareto optimal points, i.e. the optimal trade-off curve in the $(PAPR, OBP)$-reduction plane. The purpose of the factor $\mu = ||\mathbf{s}||_\infty / ||\mathbf{b}||_2$ is to ensure that the two optimization criteria are approximately equally weighted for a value of $\lambda = 0.5$.

In [2] and [3] it was observed that the unconstrained optimization problem sometimes yields a solution that allocates much more power to the reserved tones than to the corresponding number of data carriers. In this case, the transmitted signal

TABLE I
SIMULATION PARAMETERS

FFT-length	$N_c = 64$
Length of CP	$N_{cp} = 0$
Alphabet	$\mathcal{A}$: QPSK
Used Subcarriers	$\mathcal{M} = \{\pm1, \pm2, \dots, \pm25\}$
Reserved Tones	$\mathcal{M}_r = \{\pm10, \pm20, \pm24, \pm25\}$
Oversampling Factor	$L = 4$
Freq. where OBP is minimized	$\mathcal{V} = \{\pm25.9, \pm26.1, \pm26.9, \pm27.1\}$
Occupied Frequency Band	$[-25.5; 25.5]$
Average Power Constraint	$P_{\text{av}} = 0\,\text{dB}$
Maximum Power Constraint	$P_{\max} = \infty$

IFFT$\{\tilde{\mathbf{a}} + \tilde{\mathbf{x}}\}$ has to be normalized by the factor[3]

$$\alpha = \sqrt{\frac{|\mathcal{M}|}{|\mathcal{M}_d| + ||\mathbf{x}||_2^2}} \tag{11}$$

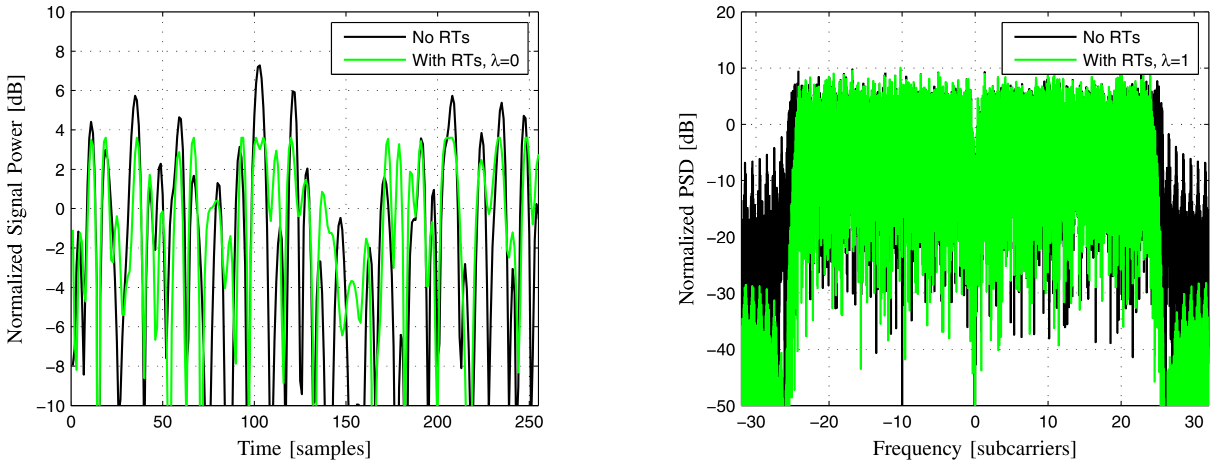
in order to constrain the total symbol energy to $|\mathcal{M}|$. Besides reducing the energy that is available for data transmission, this normalization has an additional drawback in higher-order modulation schemes like QAM or APSK, where the amplitude carries information: Since the value of α changes randomly between the OFDM symbols, the receiver experiences a randomly changing magnitude of the effective channel gain, which makes an interpolation of the effective channel magnitude in time direction impossible. This obviously leads to a degradation of the MSE of the channel estimates, especially in slowly varying channels.

In order to avoid this problem, we introduce power constraints into the optimization algorithm. The condition $||\mathbf{x}||_2^2 \le |\mathcal{M}_r| P_{\text{av}}$ ensures that on average, the allocated power per reserved tone is not higher than P_{av}. Furthermore, the power of each reserved tone can be limited with the constraint $||\mathbf{x}||_\infty^2 \le P_{\max}$, thereby preventing high peaks in the PSD.

The total optimization problem can now be stated as follows:

$$\begin{aligned} &\text{minimize}_{\mathbf{x}} && \begin{pmatrix} 1-\lambda \\ \mu\lambda \end{pmatrix}^T \begin{pmatrix} ||Q\mathbf{x} + \mathbf{s}||_\infty \\ ||A\mathbf{x} + \mathbf{b}||_2 \end{pmatrix} \\ &\text{subject to} && ||\mathbf{x}||_2^2 \le |\mathcal{M}_r| P_{\text{av}} \\ &&& ||\mathbf{x}||_\infty^2 \le P_{\max} \end{aligned} \tag{12}$$

Since the objective function as well as both constraint functions are convex, (12) is a convex optimization problem and can hence be solved by standard algorithms like the gradient descent method or Newton's method. Note that these algorithms always converge to the *global* optimum due to the convexity of the problem [4].



(a) PAPR reduction

(b) OBP reduction

Fig. 2. Effect of the PAPR reduction (a) and of the OBP reduction (b)

IV. Simulation Results

In this section, we present simulation results that show the performance of the joint optimization algorithm. The results have been obtained using the Matlab package CVX [5].

An OFDM system with 64 subcarriers is considered, 50 of them being in use ($-25, \ldots, 25$, excluding the DC-carrier). The carriers $\{\pm10, \pm20, \pm24, \pm25\}$ are chosen as reserved tones. Note that while it is important to use the outermost tones as RTs, since these contribute most to the sidelobe cancellation, the positions of the inner RTs are chosen arbitrarily, as an analysis of the optimum positions was outside the focus of this work. The frequencies $\mathcal{V}$ where the OBP reduction algorithm evaluates the PSD lie around the subcarriers ±26 and ±27. The occupied frequency band is assumed as the interval $[-25.5; 25.5]$, i.e. the power outside this interval is considered to be out-of-band power.

The CP length, needed for the calculaton of A and $\mathbf{b}$ in (12), is set to $N_{cp} = 0$, because our aim was to minimize the MAI in a cellular system with OFDMA uplink, which is caused by the OBP in the received signal *after CP removal*.

The average power allocated to the reserved tones is limited to $P_{\text{av}} = 0\,\text{dB}$, which means that the average transmit power does not exceed $|\mathcal{M}|$ and a normalization as discussed in Section III-C is unnecessary. With this constraint, the power of a single RT rarely exceeds $3\,\text{dB}$, so we chose $P_{\max} = \infty$, effectively deactivating the second constraint in (12). The simulation parameters are summarized in Table I.

In order to quantify the performance of the reduction algorithm, both the PAPR and the OBP of the signal with optimized reserved tones are compared to a reference signal where the reserved tones are randomly QPSK modulated, i.e. serve as normal data carriers.

Fig. 2 visualizes the effect of the two optimization algorithms, applied separately. Fig. 2(a) shows the ratio of instantaneous signal power to mean power for one exemplary OFDM symbol. The trade-off parameter is set to $\lambda = 0$ (full emphasis on PAPR reduction). It can be seen that the reference signal has a peak power of around $7\,\text{dB}$, whereas the optimized signal never exceeds a power of $4\,\text{dB}$, yielding a PAPR reduction of more than $3\,\text{dB}$. Fig. 2(b) shows the PSD of a signal consisting of 100 OFDM symbols, with and without reserved tones, this time optimized for OBP reduction. It can clearly be observed that the optimization algorithm leads to an OBP reduction of more than $20\,\text{dB}$.

We now turn our attention to the joint reduction of PAPR and OBP. Fig. 3(a) shows the mean reduction of both quantities as a function of the trade-off parameter λ. The achievable PAPR reduction is higher than $3\,\text{dB}$ for $\lambda = 0$, and decreases to about $-0.13\,\text{dB}$ as more and more emphasis is put onto the OBP reduction. Thus, a pure OBP reduction even leads to a slight PAPR increase. The OBP plot, on the other hand, starts at around $0\,\text{dB}$ and reaches a maximum of more than $23\,\text{dB}$ for $\lambda = 1$.

The same values are plotted in Fig. 3(b), this time as a trade-off curve in the $(PAPR, OBP)$-reduction plane. For a comparison, this figure also includes the performance that is achieved by two separate, serially concatenated optimization steps. In these simulations, the carriers $\{\pm10, \pm20\}$ are solely used for PAPR reduction, and the carriers $\{\pm24, \pm25\}$ are optimized only for sidelobe cancellation. The only degree of freedom available in this case is the order in which the two optimizations are carried out: The circle in Fig. 3(b) corresponds to the simulation where the PAPR reduction was carried out first and the OBP reduction afterwards; the cross marks the other way round. In this figure, the superiority of the joint optimization can clearly be seen.

So far, only the mean performance has been considered. To give an idea of the distribution of PAPR and OBP values, we finish this section with Figure 4, where the complementary cumulative distribution functions (CCDF) for PAPR (a) and

[3] Recall that we assume unit energy for each data carrier on average.

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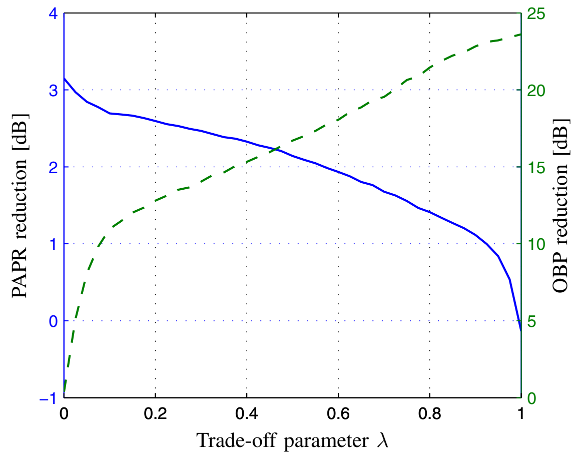
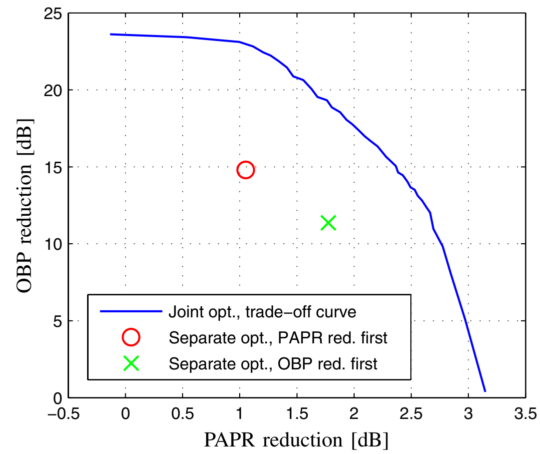
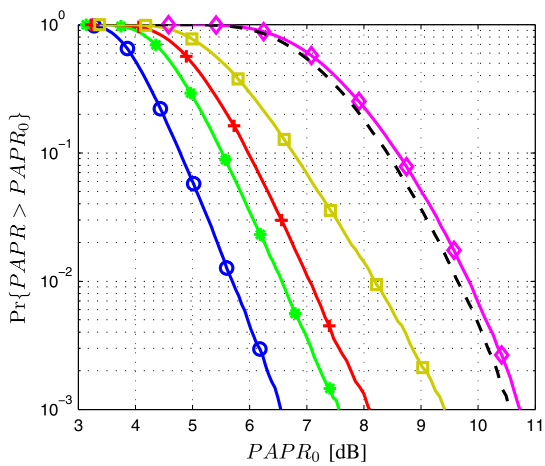


(a) Performance vs. the trade-off parameter; '——': PAPR; '– –': OBP

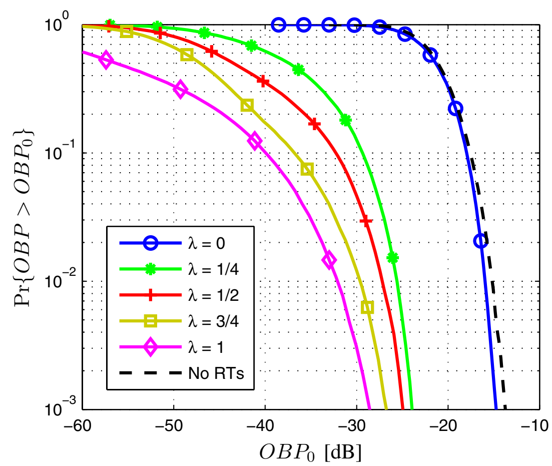


(b) Trade-off curve and points achievable with separate optimization

Fig. 3. Performance of the joint reduction algorithm



(a)



(b)

Fig. 4. CCDFs of the PAPR (a) and the OBP (b) for different values of λ. The dashed line corresponds to a signal without any reserved tones.

OBP (b) are plotted for different values of λ, as well as for the reference signal without any reserved tones. We see that even a rather low value of $\lambda = 1/4$ has a large impact on the OBP, while the PAPR reduction is only decreased by $1\,\mathrm{dB}$ at a clipping rate of 10^{-3} in comparison to $\lambda = 0$.

V. Conclusion

We have proposed to use the tone reservation technique, which has been examined in the literature separately for peak-to-average power ratio reduction and out-of-band power reduction, in order to jointly optimize both PAPR and OBP. Simulation results show the superior performance of our algorithm in comparison to a system that performs two separate optimization steps. A further advantage is the possibility to adjust the relative weighting of the two criteria dynamically. The joint reduction approach is therefore a promising technique for OFDM systems in which both a high PAPR and a high OBP are problematic, as for example in cellular mobile systems with an OFDMA-based uplink.

For a real-time implementation of the proposed algorithm, however, its relatively high computational complexity has to be considered. Therefore, algorithms with lower complexity will be investigated in future work.

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