Limited CSI Feedback based on an Adaptive Codebook for Temporally Correlated MISO Fading Channels

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***Abstract*—To increase the link capacity and reliability, the use of channel state information at the transmitter in multi-antenna systems is often discussed in current research. Such information is often obtained via a feedback link which is usually limited by rate constraints. In this paper, we propose a novel scheme to quantize the channel direction vector by exploiting the temporal correlation for MISO fading channels. With the same amount of feedback bits, our proposed scheme achieves a significant capacity gain on the forward link compared to conventional quantization techniques. This gain increases with decreasing channel dynamics.**

I. Introduction

Wireless systems using multiple antennas have attracted substantial interest of research for their potential to increase the spectral efficiency and robustness against fading. In Multiple-Input-Single-Output (MISO) systems, beamforming is an efficient technique to provide capacity gains in fading channels. However, Channel State Information (CSI) at the Transmitter (CSIT) is required for beamforming. To attain the necessary CSI for transmitter beamforming, limited feedback was first proposed in [1] where the CSI was conveyed to the transmitter via a finite rate feedback link. In order to meet this rate constraint, various techniques have been proposed for quantizing CSI such as Grassmannian line packing [2], Vector Quantization (VQ) [3] and Random Vector Quantization (RVQ) [4]. They have shown that the capacity improvement is proportional to the amount of feedback bits available. However, the penalty paid for it is the overhead that feedback introduces as it consumes bandwidth on the feedback link.

Many past works on limited feedback are based on the assumption of a block fading channel [2], [3], [4] and [5]. In order to increase the spectral efficiency of the feedback link, one intuitive solution is to exploit the temporal correlation of the fading channel. Therefore, in this paper, we study the case where the channel is continuously varying over time and the fading process is in contrast to the block fading model stationary. Some researchers have worked on feedback rate reduction for temporally correlated channels. Roh and Rao [6] utilized adaptive delta modulation to quantize smoothly changing parameters extracted from the CSI. For this algorithm, the minimum required amount of feedback bits increases with the number of transmitter and receiver antennas. Banister and Zeilder [7] proposed a stochastic gradient adaptation algorithm by using one bit feedback to maximize the received SNR. However, this requires the transmitter to broadcast information on channel subspace matrices which decrease the spectral efficiency of the forward link transmission. The approach introduced by Huang et al. in [8] reduces the amount of feedback bits by ignoring transitions between Markov states of the channel that occur with small probabilities. This algorithm is most efficient in case of a high quantization resolution.

The present work proposes a practical, low complexity quantization scheme which exploits the temporal correlation of fading channels. While maintaining the same capacity on the forward link, it can be used to decrease the amount of feedback bits compared to conventional quantization schemes. Following the same procedure of the codebook construction as in the RVQ scheme, the codebook for the proposed scheme is randomly generated by using a random number generator. Its seed is common to the transmitter and receiver. Therefore, the complexity of the codebook construction based on this scheme is much smaller than based on the VQ scheme [3]. In principle, this scheme tracks the change of the fading channel by adapting its codebook used for feedback. Numerical results reveal that it outperforms other feedback techniques whose codebooks do not track the the channel variation over time such as Grassmannian line packing and RVQ.

We start with the description of the system model in Section II. The main contribution of this paper is the adaptive quantization scheme proposed in Section III. In Section IV, its performance is studied. Numerical results are presented in Section V. We conclude the paper in Section VI.

II. System Model

We consider a $M \times 1$ MISO flat fading channel with a transmitter equipped with M antennas and a single antenna receiver. By denoting the CSI at time instance k as $\boldsymbol{h}_k = [h_{k,1}, \ldots, h_{k,M}]^T \in \mathbb{C}^{M\times 1}$, the channel input vector as $\boldsymbol{x}_k \in \mathbb{C}^{M\times 1}$ and the received signal as $y_k \in \mathbb{C}$, we get the following system input-output relationship:

$$y_k = \boldsymbol{h}_k^T \boldsymbol{x}_k + n_k \tag{1}$$

where n_k is white Gaussian noise with variance σ_n^2. The average transmit power constraint is given by $\mathcal{E}\{\boldsymbol{x}_k^\dagger \boldsymbol{x}_k\} = P$, where † represents conjugate transpose. Assuming the channel to be spatially uncorrelated and each path having unit mean power, $\boldsymbol{h}_k$ is distributed according to $\mathcal{CN}(\mathbf{0}, I_M)$ without loss of generality, where I_M indicates a $M \times M$ identity matrix. In addition, the channel vector $\boldsymbol{h}_k$ can be decomposed into two independent components in terms of its magnitude and its direction as $\boldsymbol{h}_k = \alpha_k \tilde{\boldsymbol{h}}_k$ where $\alpha_k = \|\boldsymbol{h}_k\|$ and $\tilde{\boldsymbol{h}}_k = \boldsymbol{h}_k / \|\boldsymbol{h}_k\|$. $\tilde{\boldsymbol{h}}_k$ is uniformly distributed over the M dimensional unit-norm complex space Ω_M.

Considering the temporal correlation of the fading channel $\{\boldsymbol{h}_k\}$, the autocorrelation function $R(l)$ of the fading processes $\{h_{k,i}\}$ with respect to each transmitter antenna is defined as

$$\mathcal{E}\left\{h_{k,i}^{\dagger} h_{k-l,i}\right\} = R(l), \quad \forall\, i \in [1, \ldots, M] \tag{2}$$

where l indicates the time difference.

The channel is assumed to be perfectly known at the receiver while the transmitter has B bits of channel information provided by the error free feedback link without delay. The feedback period is equal to the symbol period T. We assume the transmission strategy is beamforming. That is, an information bearing symbol $s_k \in \mathbb{C}$ is transmitted over multiple transmit antennas as $\boldsymbol{x}_k = \boldsymbol{u}_k s_k$, where $\boldsymbol{u}_k$ is a beamforming vector. Then, the received signal becomes

$$y_k = \boldsymbol{h}_k^T \boldsymbol{u}_k s_k + n_k = \alpha_k \tilde{\boldsymbol{h}}_k^T \boldsymbol{u}_k s_k + n_k. \tag{3}$$

Note that the optimal beamforming that maximizes the mutual information for MISO systems is $\boldsymbol{u}_k = \tilde{\boldsymbol{h}}_k^*$ with * indicating the conjugate, in case the transmitter has perfect knowledge of $\tilde{\boldsymbol{h}}_k$ [9]. Based on B bits feedback information, the transmitter is only able to obtain a quantized version denoted as $\boldsymbol{q}_{k,opt}$. It is appropriately selected by the receiver from a common codebook $\mathcal{Q}_k = \left\{\boldsymbol{q}_{k,1}, \ldots, \boldsymbol{q}_{k,N}\right\}$, $N = 2^B$ and its index is fed back to the transmitter. In this paper, we assume that the transmitter takes $\boldsymbol{q}_{k,opt}$ as $\tilde{\boldsymbol{h}}_k$ and then its beamforming vector $\boldsymbol{u}_k$ is set to be $\boldsymbol{q}_{k,opt}^*$. Its optimality with respect to maximizing the capacity has been discussed in [10]. To maximize the mutual information between s_k and y_k in this setting, the receiver selects the quantized channel direction vector in the codebook $\mathcal{Q}_k$ according to

$$\boldsymbol{q}_{k,opt} = \arg \max_{\boldsymbol{q}_{k,j} \in \mathcal{Q}_k} \left|\tilde{\boldsymbol{h}}_k^{\dagger} \boldsymbol{q}_{k,j}\right|^2 \tag{4}$$

where all vectors $\left\{\boldsymbol{q}_{k,j}\right\}$ in the codebook $\mathcal{Q}_k$ have unit-norm. Therefore, the average power constraint on $\boldsymbol{x}_k$ is equivalent to $\mathcal{E}\left\{|s_k|^2\right\} = P$.

III. Adaptive Vector Quantization

In this section, we develop the Adaptive Vector Quantization (AVQ) scheme for quantizing channel direction vectors. It consists of two steps: modeling the temporal correlation of the channel direction vector and constructing an adaptive codebook.

A. Temporal correlation of the channel direction vector

In typical slowly fading channels, the change of CSI between two consecutive time slots is small. Based on the knowledge of CSIT at time instance $k-1$, we only need to quantize the small variation space of the CSI at time instance k. As we did not quantize the whole region of the CSI, the quantization resolution can be significantly increased for a given feedback rate constraint. Since we restrict the transmission strategy to beamforming without power control, the transmitter only needs to know the quantized version of the channel direction vector. In order to exploit the temporal correlation of the channel direction vector, we first define the region $\mathcal{R}_k(\varepsilon, \tilde{\boldsymbol{h}}_{k-1})$ given by

$$\mathcal{R}_k(\varepsilon, \tilde{\boldsymbol{h}}_{k-1}) = \left\{\tilde{\boldsymbol{h}}_k \,\middle|\, \mid \tilde{\boldsymbol{h}}_k^{\dagger} \tilde{\boldsymbol{h}}_{k-1} \mid \geq \varepsilon, \quad \|\tilde{\boldsymbol{h}}_k\|^2 = 1\right\} \tag{5}$$

where ε is a given threshold in the range of $[0, 1]$. The size of this region $\mathcal{R}_k(\varepsilon, \tilde{\boldsymbol{h}}_{k-1})$ depends on the threshold ε, i.e., $\mathcal{R}_k(\varepsilon_1, \tilde{\boldsymbol{h}}_{k-1}) \subseteq \mathcal{R}_k(\varepsilon_2, \tilde{\boldsymbol{h}}_{k-1})$ if $\varepsilon_1 \geq \varepsilon_2$. Then, we calculate the probability of $\tilde{\boldsymbol{h}}_k \in \mathcal{R}_k(\varepsilon, \tilde{\boldsymbol{h}}_{k-1})$ denoted as $P_\varepsilon = P\left(|\tilde{\boldsymbol{h}}_k^{\dagger} \tilde{\boldsymbol{h}}_{k-1}| \geq \varepsilon \middle| \tilde{\boldsymbol{h}}_{k-1}\right)$. By decomposing $\boldsymbol{h}_k$ into the summation between $\boldsymbol{h}_k^{\parallel} = \tilde{\boldsymbol{h}}_{k-1}^{\dagger} \boldsymbol{h}_k \tilde{\boldsymbol{h}}_{k-1}$ and $\boldsymbol{h}_k^{\perp} = \boldsymbol{h}_k - \boldsymbol{h}_k^{\parallel}$, this probability P_ε becomes

$$P_\varepsilon = P\left(\frac{\|\boldsymbol{h}_k^{\parallel}\|^2}{\|\boldsymbol{h}_k^{\perp}\|^2} \geq \frac{\varepsilon^2}{1-\varepsilon^2} \middle| \tilde{\boldsymbol{h}}_{k-1}\right).$$

With $\beta = R(1)$, the $\boldsymbol{h}_k$ can be expressed as

$$\boldsymbol{h}_k = \beta \boldsymbol{h}_{k-1} + \sqrt{1-|\beta|^2}\, \boldsymbol{w}_{k-1}$$

where $\boldsymbol{w}_{k-1} \sim \mathcal{CN}(0, I_M)$ is independent to $\boldsymbol{h}_{k-1}$. Then, the parallel component $\boldsymbol{h}_k^{\parallel}$ of $\boldsymbol{h}_k$ to $\tilde{\boldsymbol{h}}_{k-1}$ can be written as

$$\begin{aligned}
\boldsymbol{h}_k^{\parallel} &= \beta \boldsymbol{h}_{k-1} + \sqrt{1-|\beta|^2}\, \tilde{\boldsymbol{h}}_{k-1}^{\dagger} \boldsymbol{w}_{k-1} \tilde{\boldsymbol{h}}_{k-1} \\
&= \beta \boldsymbol{h}_{k-1} + \sqrt{1-|\beta|^2}\, \boldsymbol{w}_{k-1}^{\parallel} \\
&\approx \beta \boldsymbol{h}_{k-1}
\end{aligned}$$

where $\boldsymbol{w}_{k-1}^{\parallel}$ is the parallel component of $\boldsymbol{w}_{k-1}$ to $\tilde{\boldsymbol{h}}_{k-1}$ and the approximation is valid because $|\beta|$ is close to unity considering typical temporal correlation of fading channels. Similarly, the orthogonal component is given by

$$\begin{aligned}
\boldsymbol{h}_k^{\perp} &= \beta \boldsymbol{h}_{k-1} + \sqrt{1-|\beta|^2}\, \boldsymbol{w}_{k-1} - \boldsymbol{h}_k^{\parallel} \\
&= \sqrt{1-|\beta|^2} \left(\boldsymbol{w}_{k-1} - \boldsymbol{w}_{k-1}^{\parallel}\right) \\
&= \sqrt{1-|\beta|^2}\, \boldsymbol{w}_{k-1}^{\perp}.
\end{aligned}$$

Therefore, the probability P_ε is approximately equal to

$$\begin{aligned}
P_\varepsilon &\approx P\left(\frac{\|\boldsymbol{h}_{k-1}\|^2}{\|\boldsymbol{w}_{k-1}^{\perp}\|^2} \geq \frac{\varepsilon^2(1-|\beta|^2)}{(1-\varepsilon^2)|\beta|^2} \middle| \tilde{\boldsymbol{h}}_{k-1}\right) \\
&= P\left(\frac{\|\boldsymbol{h}_{k-1}\|^2}{\|\boldsymbol{w}_{k-1}^{\perp}\|^2} \geq \frac{\varepsilon^2(1-|\beta|^2)}{(1-\varepsilon^2)|\beta|^2}\right)
\end{aligned}$$

where the second equality is because $\|\boldsymbol{h}_{k-1}\|^2$ and $\|\boldsymbol{w}_{k-1}^{\perp}\|^2$ are both independent to $\tilde{\boldsymbol{h}}_{k-1}$. Since $\|\boldsymbol{h}_{k-1}\|^2$ and $\|\boldsymbol{w}_{k-1}^{\perp}\|^2$ are independent χ^2-distributed random variables with $2M$ and $2(M-1)$ degrees of freedom, respectively, the ratio of them follows the F distribution. Hence, the probability P_ε is approximately equal to

$$\begin{aligned}
P_\varepsilon &\approx 1 - F_{2M, 2(M-1)}\left(\frac{M-1}{M} \cdot \frac{\varepsilon^2(1-|\beta|^2)}{(1-\varepsilon^2)|\beta|^2}\right) \\
&= 1 - I_z\left(M, M-1\right), \quad z = \frac{\varepsilon^2(1-|\beta|^2)}{\varepsilon^2(1-2|\beta|^2) + |\beta|^2}
\end{aligned} \tag{6}$$

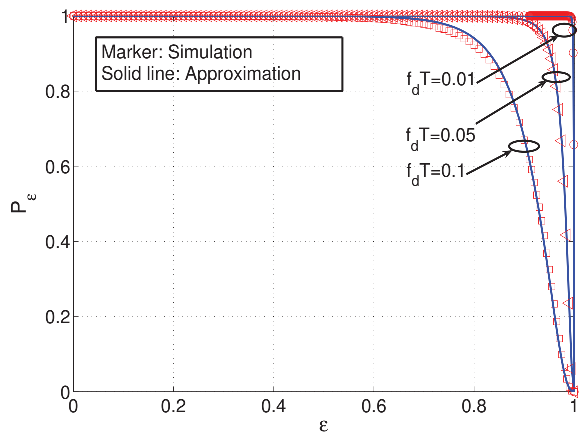

Fig. 1. Comparison of the approximation of P_ε in (6) with simulation results for $M = 4$ transmit antennas.

where $F_{m,n}(x)$ is the cumulative distribution function (CDF) of the F distribution and $I_z(a, b)$ is the regularized incomplete beta function [11]. In Fig. 1, the probability P_ε is shown over the threshold $\varepsilon \in [0, 1]$ for a fading channel whose autocorrelation function is described by a Jakes' model with a maximum Doppler frequency f_d. For f_dT smaller than 0.05 with T indicating the symbol period, typically being the case for realistic fading channels, (6) approximates the actual value of P_ε very well. There exists a high probability that the channel direction vector $\tilde{\boldsymbol{h}}_k$ at the kth time instance is located in the region $\mathcal{R}_k(\varepsilon, \tilde{\boldsymbol{h}}_{k-1})$. For instance, with $f_dT = 0.05$, P_ε almost approaches unity for the choice of $\varepsilon = 0.8$. This demonstrates the temporal correlation between two consecutive channel direction vectors. Moreover, for a given P_ε, the threshold ε is larger for more correlated fading channels, e.g., $f_dT = 0.01$.

B. Adaptive codebook construction

The observation in Fig. 1 motivates to use the feedback bits to quantize the region $\mathcal{R}_k(\varepsilon, \tilde{\boldsymbol{h}}_{k-1}) \subset \Omega_M$. Due to the lack of knowledge on $\tilde{\boldsymbol{h}}_{k-1}$ at the transmitter, we quantize the subspace $\mathcal{R}_k(\varepsilon, \boldsymbol{q}_{k-1,opt})$.

Denote the absolute value of the inner product $|\tilde{\boldsymbol{h}}_k^\dagger \boldsymbol{q}_{k-1,opt}|$ as η_k. For some $\vartheta \in [0, 2\pi)$, $\tilde{\boldsymbol{h}}_k \in \mathcal{R}_k(\varepsilon, \boldsymbol{q}_{k-1,opt})$ can be decomposed into

$$\tilde{\boldsymbol{h}}_k = e^{j\vartheta}\left(\eta_k \boldsymbol{q}_{k-1,opt} + \sqrt{1-\eta_k^2}\,\boldsymbol{v}_k\right) \tag{7}$$

where $\eta_k \geq \varepsilon$ and $\boldsymbol{v}_k$ is a unit-norm vector orthogonal to $\boldsymbol{q}_{k-1,opt}$. Note that the beamforming vector is independent of $e^{j\vartheta}$ which can be seen from (4). With the knowledge of $\boldsymbol{q}_{k-1,opt}$, only the parameters η_k and $\boldsymbol{v}_k$ are of interest.

To keep the AVQ scheme simple, its codebook construction follows the same procedure as the RVQ scheme [4], which means all elements $\{\boldsymbol{q}_{k,j}\}$ in the codebook $\mathcal{Q}_k$ are randomly generated in the region $\mathcal{R}_k(\varepsilon, \boldsymbol{q}_{k-1,opt})$. Based on the decomposition of $\tilde{\boldsymbol{h}}_k$ in (7), the new codebook elements $\boldsymbol{q}_{k,j} \in \mathcal{R}_k(\varepsilon, \boldsymbol{q}_{k-1,opt})$ can be generated according to

$$\boldsymbol{q}_{k,j} = \mu_{k,j}\boldsymbol{q}_{k-1,opt} + \sqrt{1-\mu_{k,j}^2}\,\boldsymbol{g}_{k,j} \tag{8}$$

where the variables $\mu_{k,j}$ are real valued and uniformly distributed over $[\varepsilon, 1)$. With a given $\boldsymbol{q}_{k-1,opt}$, $\boldsymbol{g}_{k,j} \in \Omega_M$ is randomly generated and can be formed as

$$\boldsymbol{g}_{k,j} = G(\boldsymbol{q}_{k-1,opt})[0, \boldsymbol{p}_{k,j}^T]^T \tag{9}$$

where $\boldsymbol{p}_{k,j}^T$ is a $M-1$ dimensional vector uniformly distributed over Ω_{M-1} and G is a unitary matrix depending on $\boldsymbol{q}_{k-1,opt}$. It can be obtained in various ways, such as using Givens rotations and Householder reflections [12].

With B quantization bits, we summarize the construction of the adaptive codebook $\{\mathcal{Q}_k\}$ as follows:

1) Initialization:
$$\mathcal{Q}_1 = \left\{\boldsymbol{q}_{1,1}, \ldots, \boldsymbol{q}_{1,2^B}\right\}, \quad \mathcal{Q}_1 \subset \Omega_M$$
where $\boldsymbol{q}_{1,j}$ is randomly selected in Ω_M.
2) For the kth time instance:
$$\mathcal{Q}_k = \left\{\boldsymbol{q}_{k,1}, \ldots, \boldsymbol{q}_{k,2^B}\right\}, \quad \mathcal{Q}_k \subset \mathcal{R}_k(\varepsilon, \boldsymbol{q}_{k-1,opt})$$
where $\left\{\boldsymbol{q}_{k,j}\right\}$ are randomly generated following (8).

The quantization metric is given in (4). The optimum choice of the parameter ε that maximizes the mutual information on the forward link depends on the channel dynamics and the amount of feedback bits B. As a closed form solution for the calculation of the optimum ε is not available, we optimize ε based on Monte Carlo simulations, see Section V.

IV. Performance Analysis

In this section, the ergodic capacity for the proposed adaptive quantization scheme is studied. Since the channel noise is complex Gaussian and CSI is perfectly known at the receiver, the capacity achieving distribution of the channel input vector $\boldsymbol{x}_k$ is proper Gaussian with the average power constraint P.

Starting from the expression of the input vector $\boldsymbol{x}_k = \boldsymbol{u}_k s_k$, $\boldsymbol{x}_k$ consists of the information bearing input symbol s_k and the beamforming $\boldsymbol{u}_k$ which is determined by the feedback information, i.e., $\boldsymbol{u}_k = \boldsymbol{q}_{k,opt}^*$. By combining the beamforming vector with the fading channel, we obtain an equivalent channel model corresponding to a SISO channel denoted as $\boldsymbol{h}_k^T\boldsymbol{u}_k$ with input signal s_k and outputs $(y_k, \boldsymbol{h}_k)$. It can be shown that the random process $\{|\boldsymbol{h}_k^T\boldsymbol{u}_k|\}$ is ergodic. The proof is omitted here due to space limitations. Thus, the ergodic channel capacity of the forward link can be obtained as

$$C = \lim_{L\to\infty} \frac{1}{L} \cdot I\left(s_1^L; y_1^L, \boldsymbol{h}_1^L \middle| \boldsymbol{f}_1^L\right) \tag{10}$$

where $I(\cdot)$ specifies the mutual information. $\boldsymbol{f}_1^L$ stands for the collection of seeds for the random number generator from the first to the Lth time instance. $\boldsymbol{f}_1^L$ is perfectly known by both the transmitter and the receiver. Due to the independence of the information signal s_k and the channel $\boldsymbol{h}_k$, (10) is equivalent to

$$\begin{aligned} C &= \lim_{L\to\infty} \frac{1}{L} \cdot \left(I\left(s_1^L; y_1^L | \boldsymbol{h}_1^L, \boldsymbol{f}_1^L\right) + I\left(s_1^L; \boldsymbol{h}_1^L | \boldsymbol{f}_1^L\right)\right) \\ &= \lim_{L\to\infty} \frac{1}{L} \cdot I\left(s_1^L; y_1^L | \boldsymbol{h}_1^L, \boldsymbol{f}_1^L\right). \end{aligned}$$

With the knowledge of $\boldsymbol{f}_1^k$ and $\boldsymbol{h}_1^k$, $\boldsymbol{u}_1^k$ is determined by the mapping function $t : \left(\boldsymbol{f}_1^k, \boldsymbol{h}_1^k\right) \to \boldsymbol{u}_1^k$. This mapping function $t(\cdot)$ is fixed and has been fully described in Section III-B. Therefore, the ergodic capacity could also be written as

$$\begin{aligned} C &= \lim_{L\to\infty} \frac{1}{L} \cdot I\left(s_1^L; y_1^L | \boldsymbol{f}_1^L, \boldsymbol{h}_1^L, \boldsymbol{u}_1^L = t\left(\boldsymbol{f}_1^L, \boldsymbol{h}_1^L\right)\right) \\ &= \lim_{L\to\infty} \frac{1}{L} \cdot \left(h\left(y_1^L | \boldsymbol{h}_1^L, \boldsymbol{u}_1^L\right) - h\left(y_1^L | s_1^L, \boldsymbol{h}_1^L, \boldsymbol{u}_1^L\right)\right) \\ &= \lim_{L\to\infty} \frac{1}{L} \sum_{k=1}^{L} \log\left(1 + \gamma \|\boldsymbol{h}_k\|^2 |\tilde{\boldsymbol{h}}_k^T \boldsymbol{u}_k|^2\right) \end{aligned} \quad (11)$$

with $\gamma = P/\sigma_n^2$.

To the best of our knowledge, it is difficult to obtain a closed form representation of (11). Instead, we give a lower bound to have a closer view on the performance of this AVQ scheme. Since we quantize the whole space Ω_M only at the initial time instance $k = 1$, the effect of this initialization on the capacity disappears over an infinite observation time interval. For the rest of the observation time, the quantization space $\mathcal{R}(\varepsilon, \boldsymbol{q}_{k-1,opt})$ is a subset of Ω_M. Therefore, in the following analysis, we focus on the capacity achieved by using the codebook $\mathcal{Q}_k \subset \mathcal{R}(\varepsilon, \boldsymbol{q}_{k-1,opt})$. In general, there are two possibilities at the kth time instance with $k > 1$.

1) $\tilde{\boldsymbol{h}}_k \in \mathcal{R}(\varepsilon, \boldsymbol{q}_{k-1,opt})$ with probability $P_{k,\varepsilon}$:
   According to (7) and (8), the absolute value of the inner product between $\tilde{\boldsymbol{h}}_k$ and $\boldsymbol{q}_{k,opt}$ is equal to
   $$\left|\tilde{\boldsymbol{h}}_k^T \boldsymbol{q}_{k,opt}^*\right| = \left|\eta_k \mu_{k,j} + \sqrt{(1-\eta_k^2)(1-\mu_{k,j}^2)}\boldsymbol{v}_k^T \boldsymbol{g}_{k,j}^*\right|$$
   where j is the chosen index in codebook $\mathcal{Q}_k$ for feedback at the time instance k. Since both $\tilde{\boldsymbol{h}}_k$ and $\boldsymbol{q}_{k,opt}$ belong to the quantization space $\mathcal{R}(\varepsilon, \boldsymbol{q}_{k-1,opt})$, both η_k and $\mu_{k,j}$ are larger than ε. Therefore, the inner product $|\tilde{\boldsymbol{h}}_k^T \boldsymbol{q}_{k,opt}^*|$ can be lower bounded by $\max(0, 2\varepsilon^2 - 1)$ which is denoted as λ.
2) $\tilde{\boldsymbol{h}}_k \notin \mathcal{R}(\varepsilon, \boldsymbol{q}_{k-1,opt})$ with probability $1 - P_{k,\varepsilon}$:
   At worst, $\tilde{\boldsymbol{h}}_k$ is orthogonal to all elements in $\mathcal{Q}_k$. Then, the inner product $|\tilde{\boldsymbol{h}}_k^T \boldsymbol{q}_{k,opt}^*|^2$ is lower bounded by zero.

By combining the lower bounds of both cases, we can lower bound (11) as follows

$$C \geq \lim_{L\to\infty} \frac{1}{L} \sum_{k=2}^{L} P_{k,\varepsilon} \log\left(1 + \gamma \|\boldsymbol{h}_k\|^2 \lambda\right). \quad (12)$$

This lower bound is rather loose for small values of ε. However, a large value of ε can be chosen in typical slowly fading channels, see Fig. 1.

We compare the lower bound in (12) with the capacity C_{no} for the case of no CSI available at the transmitter. Specifically, C_{no} is achieved by isotropically transmitting the signal over the space, i.e.,

$$C_{no} = \lim_{L\to\infty} \frac{1}{L} \sum_{k=1}^{L} \log\left(1 + \frac{\gamma}{M} \|\boldsymbol{h}_k\|^2\right).$$

Then, the capacity gain of our scheme with respective to C_{no} can be lower bounded by

$$\triangle = C - C_{no} \geq \triangle_1 - \triangle_2$$

with

$$\triangle_1 = \lim_{L\to\infty} \frac{1}{L} \sum_{k=2}^{L} P_{k,\varepsilon} \log\left(\frac{1 + \gamma \|\boldsymbol{h}_k\|^2 \lambda}{1 + \frac{\gamma}{M} \|\boldsymbol{h}_k\|^2}\right) \quad (13)$$

$$\triangle_2 = \lim_{L\to\infty} \frac{1}{L} \sum_{k=2}^{L} (1 - P_{k,\varepsilon}) \log\left(1 + \frac{\gamma}{M} \|\boldsymbol{h}_k\|^2\right). \quad (14)$$

This lower bound exposes a tradeoff on the choice of ε. On the one hand, the lower bound λ on the inner product $|\tilde{\boldsymbol{h}}_k^T \boldsymbol{q}_{k,opt}^*|$ in $\triangle_1$ becomes larger with the increased ε. On the other hand, a larger value of ε increases the term $\triangle_2$ due to the decreased probability $P_{k,\varepsilon}$. As it has been stated in previous section, the probability $P_{k,\varepsilon}$ depends on the dynamics of the fading channel. For a certain value of ε, it is larger in case of slower fading. Based on this observation, the gain achieved by the AVQ scheme is more significant in slowly fading channels.

V. Numerical Results

Throughout this section, we evaluate the performance of the AVQ scheme by using $M = 3$ transmit antennas. The autocorrelation of the fading channel is assumed to be described by a Jakes' model with maximum Doppler frequency f_d. For typical fading channels in mobile cellular environment, $f_d T$ is smaller than 0.05.

In Fig. 2, we compare the capacity gain achieved by AVQ towards RVQ and Grassmannian line packing under the assumption of $B = 1$ quantization bit and $\varepsilon = 0.95$. As a reference, we show the capacity for cases that the transmitter has CSI and has no CSI. The gap in between demonstrates the maximum achievable capacity gain by using CSIT. First, the AVQ outperforms RVQ and Grassmannian line packing [2] in terms of capacity. Second, as clearly reflected by the group of dashed lines with markers, the capacity gain by using the AVQ is significant especially for small f_d. This is because much more benefit of the temporal correlation can be exploited in fading channels with smaller dynamics. Note that the capacity achieved by RVQ or Grassmannian line packing is independent of the channel dynamics, since the codebook of the RVQ is generated independently over time without tracking the variation of the fading channel and Grassmannian line packing is based on a fixed codebook [2].

The numerical results in Fig. 2 have demonstrated the effectiveness of the AVQ in exploiting the temporal correlation. By properly decreasing the updating frequency in typical slowly fading channels, the feedback rate may be further reduced but with a performance degradation. In Fig. 3, we increase the feedback period from one symbol period T to T_s. During one feedback period T_s, the beamforming vector is not changed at the transmitter. As it can be seen, the achieved capacity gain is decreased with an increasing feedback period T_s.

With respect to different values of ε, the capacity achieved by using $B = 1$ feedback bit is shown in Fig. 4. As it can be seen, there exists an optimum value of ε which leads to the maximum capacity. It effectively reflects the tradeoff on the choice of ε shown in (13) and (14). The optimum value of ε is larger in case of a stronger temporal correlation of the fading

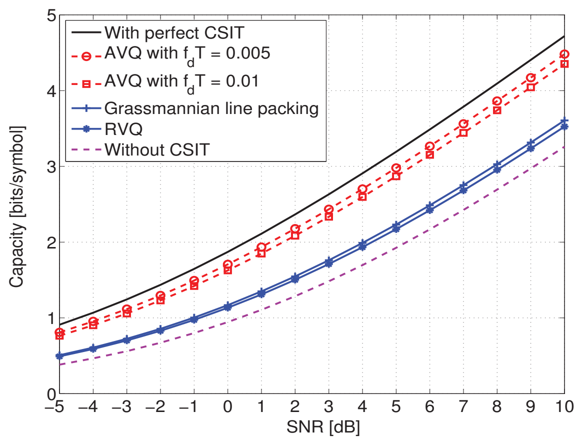

Fig. 2. Capacity achieved by AVQ, RVQ and Grassmannian line packing, M=3, $\varepsilon = 0.95$ and B=1 bit

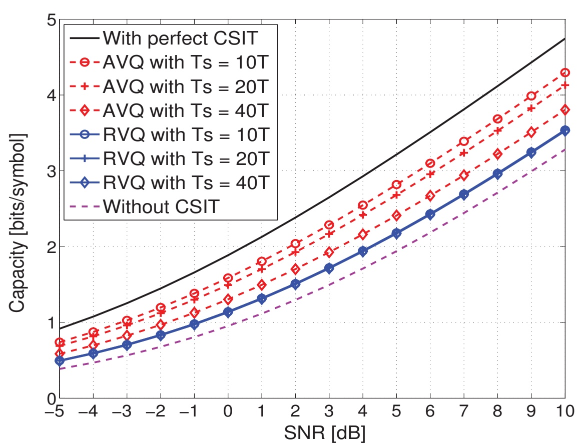

Fig. 3. Capacity achieved by using different feedback periods T_s, f_dT=0.001, $\varepsilon = 0.9$ and B=1 bit. As the difference of the RVQ performance for different T_s is small for a small amount of quantization bits, their curves overlap.

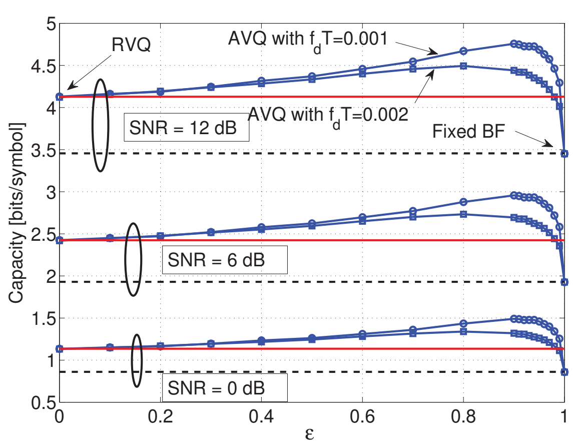

Fig. 4. The capacity achieved by using different $\varepsilon \in [0, 1]$, $B = 1$ bit, $T_s = 20T$

channel, e.g., $f_dT = 0.001$. Moreover, the difference between the maximum and minimum capacity over ε is much smaller with $f_dT = 0.002$ than $f_dT = 0.001$. It demonstrates the AVQ is much more efficient in slowly fading channels. ε is the indicator of how much temporal correlation is exploited in the AVQ. With $\varepsilon = 0$, the property of the temporal correlation is essentially neglected in the construction of the adaptive codebook. Therefore, the capacity for $f_dT = 0.001$ and $f_dT = 0.002$ converge to the same value, which is also achieved by the RVQ. If $\varepsilon = 1$, the transmitter has to use $\boldsymbol{q}_{1,opt}$ for the whole transmission time. The capacity is significantly degraded by using this outdated channel direction vector and it is the same as the case by using fixed beamforming. The above observation holds for all the SNRs shown in Fig. 4.

VI. Conclusion

We have investigated a novel scheme to quantize the channel direction vector for transmitter beamforming in a MISO system. The codebook design for feedback is based on an AVQ exploiting the temporal correlation of fading channels. The performance in terms of the capacity have been evaluated based on Monte Carlo simulations and an analytical lower bound has been given. Our numerical results indicate this proposed scheme outperforms conventional quantization techniques by exploiting the temporal correlation of the channel.

References

[1] J. Heath, R. W. and A. Paulraj, "A simple scheme for transmit diversity using partial channel feedback," in *Proc. 32nd Annu. Asilomar Conf. on Sig., Sys., and Comp.*, vol. 2, Pacific Grove, CA, USA, Nov. 1998, pp. 1073–1078.

[2] D. J. Love, J. Heath, R. W., and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2735–2747, Oct. 2003.

[3] V. Lau, Y. Liu, and T. A. Chen, "On the design of MIMO block-fading channels with feedback-link capacity constraint," *IEEE Trans. Commun.*, vol. 52, no. 1, pp. 62–70, Jan. 2004.

[4] W. Santipach and M. L. Honig, "Asymptotic performance of MIMO wireless channels with limited feedback," in *Proc. IEEE Military Commun. Conf., (MILCOM)*, vol. 1, Boston, MA, USA, Oct. 2003, pp. 141–146.

[5] K. K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On beamforming with finite rate feedback in multiple-antenna systems," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2562–2579, Oct. 2003.

[6] J. C. Roh and B. D. Rao, "An efficient feedback method for MIMO systems with slowly time-varying channels," in *Proc. IEEE Wireless Commun. and Networking Conf., (WCNC)*, vol. 2, Atlanta, GA, USA, Mar. 2004, pp. 760–764.

[7] B. C. Banister and J. R. Zeidler, "A simple gradient sign algorithm for transmit antenna weight adaptation with feedback," *IEEE Trans. Signal Process.*, vol. 51, no. 5, pp. 1156–1171, May 2003.

[8] K. Huang, B. Mondal, R. W. Heath, and J. G. Andrews, "Multi-antenna limited feedback for temporally-correlated channels: feedback compression," in *Proc. IEEE Global Telecommunications Conf., (GLOBECOM)*, San Francisco, CA, Nov./Dec. 2006, pp. 1–5.

[9] D. Tse, and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge, U.K.: Cambridge University Press, 2005.

[10] S. A. Jafar and A. Goldsmith, "On optimality of beamforming for multiple antenna systems with imperfect feedback," in *Proc. IEEE Int. Symp. on Inf. Theory, (ISIT)*, Washington, DC, June 2001, p. 321.

[11] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE J. on Sel. Areas Commun.*, vol. 24, no. 3, pp. 528–541, Mar. 2006.

[12] J. C. Roh and B. D. Rao, "Channel feedback quantization methods for MISO and MIMO systems," in *Proc. 15th IEEE Int. Symp. on Personal, Indoor and Mobile Radio Commun., (PIMRC)*, vol. 2, Paris, France, Sept. 2004, pp. 805–809.