Biological Hydrodynamics

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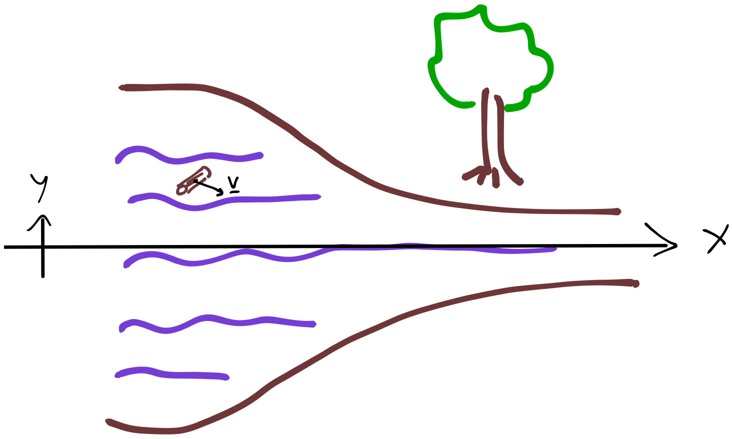
Next Tutorial: **WEDNESDAY** 30th October, 9:20 - 10:50, MPI PKS Seminar Room 3

Tutorial 2: Convective derivative



Consider a river that is flowing through a narrowing channel. Flow is steady and does not change with time, and the river shore is far away from the origin of our coordinate system. Flow at position (x, y) is specified by:

$$\underline{v} = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} kx \\ -ky \end{pmatrix}. \tag{1}$$

1. Sketch the streamlines for this 2D flow, for $x > 0$.
2. Consider a small log that is placed at different locations, and that flows with the river. Calculate the acceleration of the log as a function of position.
3. The river has been polluted with oil. For the region where $x > 0$ and $y > 0$, the concentration of oil is given by $c(x, y, t) = \alpha\, x^2\, y\, \mathrm{e}^{-kt}$, where α is a constant. If a

small log is flowing in this region of the polluted river, does the concentration of the pollutant change with time in the direct vicinity of the log?

4. For three dimensions, Stokes' theorem states that the flux through a closed surface A

$$\oint c\underline{v}\,\mathrm{d}\underline{A} \tag{2}$$

is equal to the integral over the volume V enclosed by A

$$\int \underline{\nabla}(c\underline{v})\,\mathrm{d}V\,. \tag{3}$$

Write down explicitly Stokes' theorem for the case of two and one dimensions.