On the Analysis of an Up/Down Power Controlled CDMA System in a Fast Rayleigh-Fading Environment

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***Abstract*— In this paper, we derive an analytical model of the uplink power control of a CDMA system in the presence of fast Rayleigh fading. This analysis is based on an up/down power control algorithm, as it is used for the current 3GPP standard. A stochastic nonlinear model of the closed loop power control system is derived based on statistical linearization. Our model also accounts for the effect due to the coupling of the power control loops of different users by interference and thus provides a general framework to study the effect of several user parameters like the speed of the mobiles, their target SINRs, their spreading gains, their power control stepsizes, their transmit power limits and the number of users on the SINR control errors, the transmit powers and consequently on the whole power controlled system in terms of the achieved user capacity. Our analytical results exhibit excellent accuracy shown by comparison with numerical results achieved by means of Monte Carlo simulations. In comparison to the widely studied static scenarios we are able to make statements about the system capacity in terms of supportable users, depending on the system dynamic. The results delivered by our model clearly show that with increasing channel dynamics, the number of users that can be supported with given data rates will significantly decrease.**

I. Introduction

Effective transmitter power control is essential for high-capacity cellular radio systems, to provide a satisfactory quality of service (QoS) and to cope with the near far problem. The QoS is determined by the achieved signal to noise and interference ratio (SINR). On the uplink the transmit power of the mobiles is controlled by the base station aiming each user to achieve its required SINR. In the uplink of UMTS [4], these control actions are implemented by sending commands from the base station to the mobile in order to increase or decrease its transmit power by a fixed amount. We will refer to this kind of power control as *up/down* power control. In this type of power control, the base station measures the SINR for each user and compares it with its respective target SINR. A power control bit is then sent to the mobile station once every power control group (PCG). This power control scheme works linkwise independent as the power control of each link is only based on its own SINR.

In this paper we analyze the reverse link of a CDMA system using this up/down power control algorithm in the presence of Rayleigh fast fading. There has been a lot of literature concerning the optimal transmit powers, the convergence of iterative algorithms towards this optimum and the user capacity in the case of a static scenario e.g. [9], [7], [5], [6], [8] and [1]. In practical systems the power control works in the environment of highly dynamic channels. The performance of power control in the case of realistic dynamical scenarios has only been studied by few. Some aspects of the dynamic behavior of the power control scheme have already been studied in [2], where one independently power controlled link has been examined, depending on the mobiles' velocity and the power control step size. The interference caused by other users has been treated as a constant with the reasoning that these links are also power controlled. This assumption is valid only for a certain range of velocities, due to the effect that for high velocities power control becomes worse. The main contribution of this paper is to consider the interference noise power as a stochastic process, depending on the number of users and their system parameters. Furthermore [2] only considers the variance of the power control error. We extend this model regarding to the mean values of several processes, especially of the control error and the transmit power. This enables us to give statements about feasibility and the required mobile transmit powers to achieve a specific data rate. Thus this paper enables detailed power control system analysis with regard to the interference limitation of the system considering the channel dynamics of the individual links, which is a new approach. Furthermore our model includes also the behavior of the limiter, thus showing the effect on the system if mobiles are running into their maximum transmit power. Numerical results show the accuracy of our model, which allows to study the feasibility of a given load situation concerning the given power control scheme without going through lengthy simulations.

The paper is organized as follows. After introducing the system model of the coupled power control loops, its nonlinear components are linearized. Based on this linearized model the equations for a second order analysis are derived. Finally the validity of our model is proofed by a comparison with system simulation results.

II. System Model

We consider a CDMA uplink with N users sharing the same channel. Using a flat fading model and considering the observation interval to be infinite, the received signal is given by

$$r(t) = \sum_{n=1}^{N} \sum_{j=-\infty}^{\infty} \sqrt{p_n(t)} b_{n,j} c_{n,j} s(t - jT_c - \tau_n) e^{j\phi_n} + n(t). \quad (1)$$

In this equation

- T_c is the chip duration
- $c_{n,j} \in \{-1, +1\}$ with equal probability is the value of the jth chip of the nth user
- $b_{n,j} \in \{-1, +1\}$ with equal probability represents the value of the bit containing the jth chip $c_{n,j}$. It takes on the same value for M_n successive chips where M_n is the spreading factor of user n.
- $s(t)$ is the chip waveform, which is assumed to be equal for all users, with variance $\sigma_s^2 = 1$.
- ϕ_n is the phase offset and τ_n the propagation delay of the signal of user n.
- $p_n(t)$ is the received power of user n which is equal to $a_n(t) \cdot x_n(t)$, if $a_n(t)$ is the link gain, resulting from Rayleigh fading, and $x_n(t)$ the mobile transmit power.
- $n(t)$ is a zero mean complex valued Gaussian noise process with the variance σ_n^2.

Making the same assumptions as in [2], of uniformly distributed $\tau_k \in [0, T_c]$ and $\phi_n = 0$ [1], this model yields the following slotwise equation for the SINR of user n, see [2],

$$\gamma_n(k) = \frac{p_n(k) M_n}{\sum_{m \neq n}^{N} p_m(k) + \sigma_n^2/2} = \frac{a_n(k) x_n(k) M_n}{\sum_{m \neq n}^{N} a_m(k) x_m(k) + \sigma_n^2/2}. \quad (2)$$

[1] in [2] it has been shown that this assumption has only little impact on the analysis

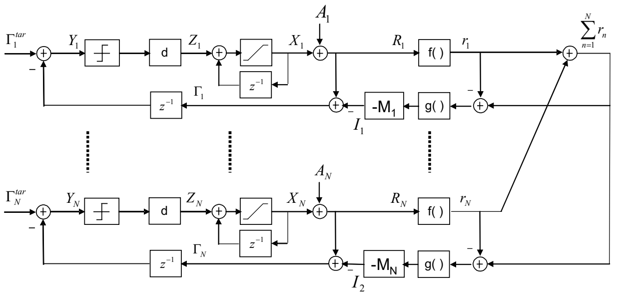


Fig. 1. Global system model

The mobile transmit power is constant over one slot. Since we want to establish a slotwise system model, also the link gain in this model has to be represented by a value for each slot. Due to the circumstance that the power is updated in dB steps, it is appropriate to use a system model in the logarithmic domain. Using instead a system model in the linear domain the power update step would depend on the actual transmit power, and thus the model would become more complex. Transforming equation (2) into the logarithmic domain leads to

$$\Gamma_n(k) = A_n(k) + X_n(k) - I_n(k) \tag{3}$$

where the capital letters denote values in dB. $I_n(k)$ is the interference term for user n, which is equal to

$$I_n(k) = 10\log_{10}\left(\frac{\sum_{m\neq n}^{N} a_m(k)x_m(k) + \sigma_n^2/2}{M_n}\right). \tag{4}$$

At the base station $\Gamma_n(k)$ is compared to the target SINR Γ_n^{tar} for each user. If $\Gamma_n > \Gamma_n^{tar}$, the base station will command the mobile to reduce its power by d dB; if instead $\Gamma_n < \Gamma_n^{tar}$ a command to increase the transmit power by d dB will be transmitted to the mobile. As Γ_n^{tar} is only adapted very slowly by an outer loop to guarantee a certain QoS requirement we can treat it as a fixed reference.

Due to the limited transmit power the mobile can not always react as it is decided by the base station. E.g. in the case the mobile transmits already with its maximal transmit power, a further power up command will not change the transmit power. Fig. 1 shows the resulting system model. The input processes to the nonlinear system $A_n(k)$ are the link gain sequences transformed into the logarithmical domain. The functions $f(x) = 10^{\frac{x}{10}}$ and $g(x) = 10\cdot\log_{10}(x+\sigma_n^2/2)$ are transformations from the logarithmic domain to the linear domain and back, including the AWGN in the back transformation $g(x)$. From here on we assume a feedback delay of one PCG. The derivation of the analytical model is straight forward in the case of larger feedback delays.

III. Statistical Linearization

The performance of the power control system can be evaluated by the distribution of the SINR error $Y_n(k) = \Gamma_n^{tar} - \Gamma_n(k-1)$. Good power control performance leads to a narrow pdf of $Y_n(k)$ with zero mean. Unfortunately, it is difficult to derive the pdf itself, and can only be achieved by solving the Fokker-Planck-Kolmogorov equation [10], which often turns out to be difficult to solve. Thus we will restrict the examination to a second order analysis of all processes. The system performance will then be evaluated on the mean and the variance of the SINR error Y_n and the transmit power X_n. For the derivation of these statistical quantities we linearize the nonlinear components like the nonlinear decision device (refered to as slicer) and limiter. Therefore we use the technique of statistical linearization [11] as it has already been used in [2] for the slicer.

A. Slicer

In contrast to [2] we extend the slicer linearization to a non zero mean input process Y_n. As it has been done in [2], we substitute the slicer by a constant gain K_n and additive white noise $W_n(k)$. The output $Z_n(k)$ of the slicer is approximated by $\hat{Z}_n(k) = K_nY_n(k) + W_n(k)$ with the constraint that Z and $\hat{Z}$ have the same mean and variance. Furthermore we choose the constant K_n to minimize the MSE $E\{(Z_n - \hat{Z}_n)^2\}$.

At this point we make the assumption that $Y_n(k) \sim N(\mu_{Y_n}, \sigma_{Y_n}^2)$, i.e. a random variable with mean μ_{Y_n} and variance $\sigma_{Y_n}^2$, as in [2]. Its validity will be discussed in section V. Assuming $\sigma_{Y_n}^2$ to be known, we shall find the optimal K_n, μ_{W_n} and $\sigma_{W_n}^2$ as a function of μ_{Y_n} and $\sigma_{Y_n}^2$. Therefore we use the following three equations:

$$E\{\hat{Z}_n\} \equiv E\{Z_n\} = \mu_{Z_n} \tag{5}$$

$$E\{\hat{Z}_n^2\} \equiv E\{Z_n^2\} = \mu_{Z_n}^2 + \sigma_{Z_n}^2 \tag{6}$$

$$0 \equiv \left.\frac{\partial E\{(Z_n - \hat{Z}_n)^2\}}{\partial K_n}\right|_{K_n = K_n^*}. \tag{7}$$

Based on the Gaussian assumption of Y_n the mean and the variance of Z_n and the correlation of Y_n and Z_n calculate to

$$\mu_{Z_n} = d\cdot\left[2\Phi\left(\frac{\mu_{Y_n}}{\sigma_{Y_n}}\right) - 1\right] \tag{8}$$

$$\sigma_{Z_n}^2 = 4d^2\cdot\left[\Phi\left(\frac{\mu_{Y_n}}{\sigma_{Y_n}}\right) - \Phi^2\left(\frac{\mu_{Y_n}}{\sigma_{Y_n}}\right)\right] \tag{9}$$

$$E\{Y_nZ_n\} = d\sqrt{\frac{2}{\pi}}\sigma_{Y_n}e^{-\frac{\mu_{Y_n}^2}{2\sigma_{Y_n}^2}} + d\mu_{Y_n}\left[\Phi\left(\frac{\mu_{Y_n}}{\sigma_{Y_n}}\right) - \Phi\left(-\frac{\mu_{Y_n}}{\sigma_{Y_n}}\right)\right] \tag{10}$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{t^2}{2}}dt$ is the Gaussian error function. Based on (5) - (7), the parameters for the linearization of the slicer are given by

$$K_n^* = \frac{1}{\sigma_{Y_n}^2}\left(E\{Y_nZ_n\} - \mu_{Y_n}\cdot\mu_{Z_n}\right) \tag{11}$$

$$\mu_{W_n} = \mu_{Z_n} - K_n^*\cdot\mu_{Y_n} \tag{12}$$

$$\sigma_{W_n}^2 = \sigma_{Z_n}^2 - {K_n^*}^2\cdot\sigma_{Y_n}^2. \tag{13}$$

B. Limiter

In the same way as we have linearized the slicer we are going to linearize the limiter. The output of the limiter $X_n(k)$ is approximated by $\hat{X}_n(k) = K_{L_n}U_n(k) + V_n(k)$ with the constraint that X and $\hat{X}$ have the same mean and variance. We assume that $U_n(k)$, the input to the limiter is Gaussian distributed and use the following equations to evaluate the constant gain K_{L_n} and the mean and the variance of $V_n(k)$:

$$E\{\hat{X}_n\} \equiv E\{X_n\} = \mu_{X_n} \tag{14}$$

$$E\{\hat{X}_n^2\} \equiv E\{X_n^2\} = \mu_{X_n}^2 + \sigma_{X_n}^2 \tag{15}$$

$$0 \equiv \left.\frac{\partial E\{(X_n - \hat{X}_n)^2\}}{\partial K_{L_n}}\right|_{K_{L_n} = K_{L_n}^*}. \tag{16}$$

Based on the Gaussian assumption of U_n the first and second moment of X_n calculate to

$$\begin{aligned}\mu_{X_n} = {} & (X_n^{min} - \mu_{U_n})\cdot\Phi\left(Q_n^{min}\right) + (X_n^{max} - \mu_{U_n})\cdot\Phi\left(-Q_n^{max}\right) \\ & + \mu_{U_n} - \frac{\sigma_{U_n}}{\sqrt{2\pi}}\left[e^{-\frac{1}{2}{Q_n^{max}}^2} - e^{-\frac{1}{2}{Q_n^{min}}^2}\right]\end{aligned} \tag{17}$$

$$\begin{aligned}E\{X_n^2\} = {} & \mu_{U_n}^2 + \sigma_{U_n}^2 + ({X_n^{min}}^2 - \mu_{U_n}^2 - \sigma_{U_n}^2)\cdot\Phi\left(Q_n^{min}\right) \\ & + ({X_n^{max}}^2 - \mu_{U_n}^2 - \sigma_{U_n}^2)\cdot\Phi\left(-Q_n^{max}\right) \\ & - \frac{\sigma_{U_n}}{\sqrt{2\pi}}\left[\mu_{U_n} + X_n^{max}\right]e^{-\frac{1}{2}(Q_n^{max})^2} \\ & + \frac{\sigma_{U_n}}{\sqrt{2\pi}}\left[\mu_{U_n} + X_n^{min}\right]e^{-\frac{1}{2}(Q_n^{min})^2}\end{aligned} \tag{18}$$

with $Q_n^{min} = \frac{X_n^{min} - \mu_{U_n}}{\sigma_{U_n}}$ and $Q_n^{max} = \frac{X_n^{max} - \mu_{U_n}}{\sigma_{U_n}}$ where X_n^{min} and X_n^{max} are the minimum and maximum transmit power of mobile n. For the correlation between U_n and X_n we get:

$$E\{X_n U_n\} = (\mu_{U_n}^2 + \sigma_{U_n}^2) + (X_n^{min} \cdot \mu_{U_n} - \mu_{U_n}^2 - \sigma_{U_n}^2) \cdot \Phi\left(Q_n^{min}\right) + (X_n^{max} \cdot \mu_{U_n} - \mu_{U_n}^2 - \sigma_{U_n}^2) \cdot \Phi\left(-Q_n^{max}\right) - \frac{\sigma_{U_n}}{\sqrt{2\pi}} \mu_{U_n} \left[e^{-\frac{1}{2} Q_n^{max^2}} - e^{-\frac{1}{2} Q_n^{min^2}} \right]. \quad (19)$$

Using these quantities in the following equations, which result from (14) - (16), we get the parameters for the linearization of the limiter

$$K_{L_n}^* = \frac{1}{\sigma_{U_n}^2} \left(E\{X_n U_n\} - \mu_{U_n} \cdot \mu_{X_n} \right) \quad (20)$$

$$\mu_{V_n} = \mu_{X_n} - K_{L_n}^* \cdot \mu_{U_n} \quad (21)$$

$$\sigma_{V_n}^2 = \sigma_{X_n}^2 - K_{L_n}^{*^2} \cdot \sigma_{U_n}^2. \quad (22)$$

C. Interference Noise Power Approximation

It remains to find an approximation for the mean and variance of the interference noise power of user n, see (4). That means that we have to find an equation for the mean and the variance of the sum of the received signal powers $R_m(k)$ $m \neq n$ of the other users and the additive white Gaussian noise power, which is treated as a constant.

Assuming that the received powers $R_m(k)$ and the interference noise power $I_n(k)$ are Gaussian distributed, we can use the *Fenton-Wilkinson (FW)* approximation, see [3], for the calculation of the mean and variance of the sum process $I_n(k)$. This assumption is an approximation, which becomes critical in case of high velocities. The *FW* transformation is derived by matching the first two moments of $\sum_{i=1}^K e^{\xi_i} \equiv e^{\chi}$ leading to

$$u_1 = E\{e^{\chi}\} = \sum_{i=1}^{K} e^{\mu_{\xi_i} + \frac{1}{2}\sigma_{\xi_i}^2} \quad (23)$$

$$u_2 = E\{e^{2\chi}\} = \sum_{i=1}^{K} e^{2\mu_{\xi_i} + 2\sigma_{\xi_i}^2} + 2 \sum_{i=1}^{K-1} \sum_{j=i+1}^{K} e^{\mu_{\xi_i} + \mu_{\xi_j}} e^{\frac{1}{2}(\sigma_{\xi_i}^2 + \sigma_{\xi_j}^2)}. \quad (24)$$

In our case the mean and the standard deviation are given in decibels, thus we have to use the following substitutions

$$\mu_{\xi_i} = \begin{cases} \frac{\ln(10)}{10} \mu_{R_i} & i = 1 \ldots N-1 \\ \ln(10) \cdot \log_{10}\left(\frac{\sigma_n^2}{2}\right) & i = N \end{cases} \quad (25)$$

$$\sigma_{\xi_i}^2 = \begin{cases} \left(\frac{\ln(10)}{10}\right)^2 \sigma_{R_i}^2 & i = 1 \ldots N-1 \\ 0 & i = N \end{cases}. \quad (26)$$

Solving (23) and (24) leads to

$$\mu_{I_n} = \frac{10}{\ln(10)} \left[2\ln(u_1) - \frac{1}{2}\ln(u_2) \right] - 10\log_{10}(M_n) \quad (27)$$

$$\sigma_{I_n}^2 = \left(\frac{10}{\ln(10)} \right)^2 \cdot \left[\ln(u_2) - 2\ln(u_1) \right] \quad (28)$$

for the mean and the variance of the interference noise power. The term $-10\log_{10}(M_n)$ results from (4).

This approximation assumes that the processes R_i are uncorrelated. Indeed, it can be shown by simulations that there are only weak correlations between the R_i's.

The whole system analysis is based on spectrum integration (see section IV), hence we need not only the variance of I_n but also its spectrum. We obtain the spectrum of I_n by calculating the autocovariance functions $C_{R_i R_i}$ of all the processes R_i, then calculate the FW approximation for each entry of the autocovariance function and thus get the autocovariance function $C_{I_n I_n}$ of the interference noise power. Based on this autocovariance function we can calculate the spectrum S_{I_n} of the interference noise power.

At this point it should be underlined that the coupling of the individual power control loops is contained in our analytical model by the calculation of the mean values and the spectra of the interference noise power processes, and thus accounts for the power control performance of the individual links.

a) Evaluation of FW-Approximation quality: As already stated in [3] the quality of the FW-approximation strongly depends on the values of the standard deviation of the processes to be summed. Furthermore we have made the assumption that the processes R_i are Gaussian distributed, which does not hold exactly, especially for high velocities. Thus we have compared analytical results based on the FW-approximation with simulation results in Fig. 2. This comparison shows, that the mean is overestimated and the variance is overestimated too in the case of high velocities. In the case all loops are parameterized in the same way and thus all the processes to be added have the same statistics, it can be shown that, decreasing the variance of all addends, the mean and the variance of the FW-approximation result also decreases. Therefore we introduce a constant $0 < \alpha < 1$ reducing the input variance (also entries of the autocovariance functions $C_{R_i R_i}$). Fig. 2 shows the parameters μ_{I_n} and $\sigma_{I_n}^2$ of the FW approximation depending on α compared to the simulation results in the case all power control loops are parameterized identically. Due to the fact that especially an overestimation of μ_{I_n} is critical due to the feedback in the system equations, we decide to choose $\alpha = 0.5$ which leads to a good match between the analysis and the simulation results for a wide range of parameters.

IV. System Equations - Spectrum Integration

Now, as we have found approximations for the nonlinear expressions, we are able to derive equations for the first and the second order moments of the various processes. Fig. 3 shows one individual power control loop of the linearized model.

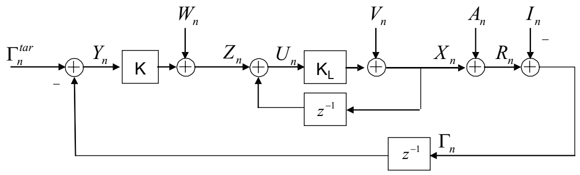


Fig. 3. Linearized Power Control Loop

The equations describing the mean values can easily be deduced from Fig. 3. We get

$$\mu_{Y_n} = \Gamma_n^{tar} - \mu_{X_n} - \mu_{A_n} + \mu_{I_n} \quad (29)$$

$$\mu_{U_n} = \mu_{Z_n} + \mu_{X_n}. \quad (30)$$

Together with (27), (8) and (17) we have found the set of equations describing the mean values.

For the calculation of the variance of the different processes we use the spectrum integration approach as it has already been used in [2]. With the linearized model of each power control loop, we can establish transfer functions between the various input and output ports. The model has four input processes, A_n, I_n, W_n and V_n, assuming Γ_n^{tar} to be constant. The processes W_n and V_n are white Gaussian noise processes, thus their spectra are completely characterized by $\sigma_{W_n}^2$ and $\sigma_{V_n}^2$, which are given by (13) and (22). The spectrum S_{I_n} is given by the FW-approximation based on μ_{R_i} and S_{R_n}. The spectrum S_{A_n} of the channel power process in the logarithmical domain, corresponding to a Rayleigh channel, is precalculated by means of Monte Carlo simulations of the channel processes for various velocities. As output processes of each power control loop we use X_n, Y_n, U_n and R_n. The variances of all those processes are required to solve the equations of the analytical system model or for system performance evaluation. From R_n we also need

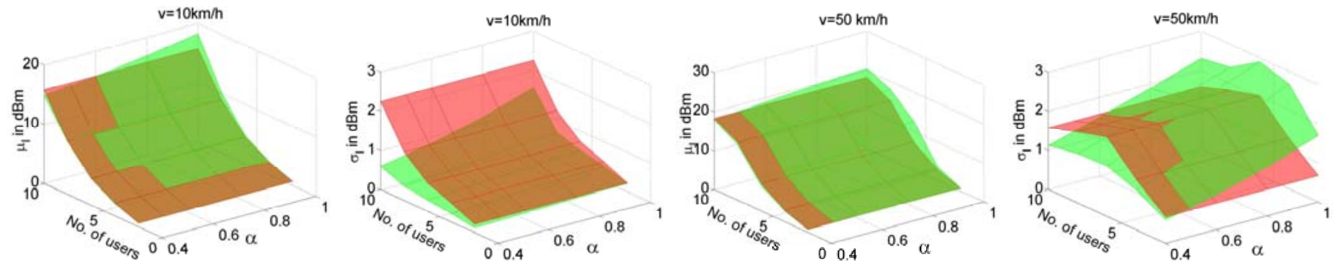


Fig. 2. FW-approximation results (green/light gray) vs. simulation (red/dark gray, independent of α) for μ_I and σ_I of the interference noise power

the spectrum as input to the FW-approximation. Assuming the four input processes to be mutually uncorrelated, which can be approximately shown for A_n and I_n by simulations (W_n and V_n are uncorrelated to the other processes by construction), the spectra of the output processes can be calculated as follows

$$\begin{aligned} S_{P_n} &= |H_{AP_n}(\omega)|^2 \cdot S_{A_n} + |H_{IP_n}(\omega)|^2 \cdot S_{I_n} \\ &\quad + |H_{WP_n}(\omega)|^2 \cdot \sigma^2_{W_n} + |H_{VP_n}(\omega)|^2 \cdot \sigma^2_{V_n} \end{aligned} \tag{31}$$

with e.g. $H_{AP_n}(\omega)$ as the transfer function of the linearized model from the input port of the process A to the output port of the process P, where P is a placeholder for the specific process. The transfer functions depend on the linearization constants K_n^* and $K_{L_n}^*$. The variances are then given by

$$\sigma^2_{P_n} = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{P_n}(\omega) d\omega. \tag{32}$$

Now we have completed our set of equations, which enables us to calculate the first and second order moments of all processes.

V. Numerical Results

Now all the equation of the analytical model have been found. In order to obtain numerical results, in this section the nonlinear set of equations ((8), (9), (10), (11), (12), (13), (17), (18), (19), (20), (21), (22), (23), (24), (25), (26), (27), (28), (29), (30), (31) and (32)) has been solved numerically for some scenarios using MATLAB and the results have been compared to simulation results. The simulation model includes the power control loop implementations for all users, including the nonlinearities, and also the interaction of these users as interference to each other.

For all evaluated scenarios we assume a transmit power dynamic range of 65dB for all mobiles according to [4]. Furthermore we assume that respectively to a target SIR of 0dB the transmit power can increased about 21dB and can be decreased about 44dB. Since we are only using relative values and no absolute values, it is not necessary to make further assumptions about path loss, etc..

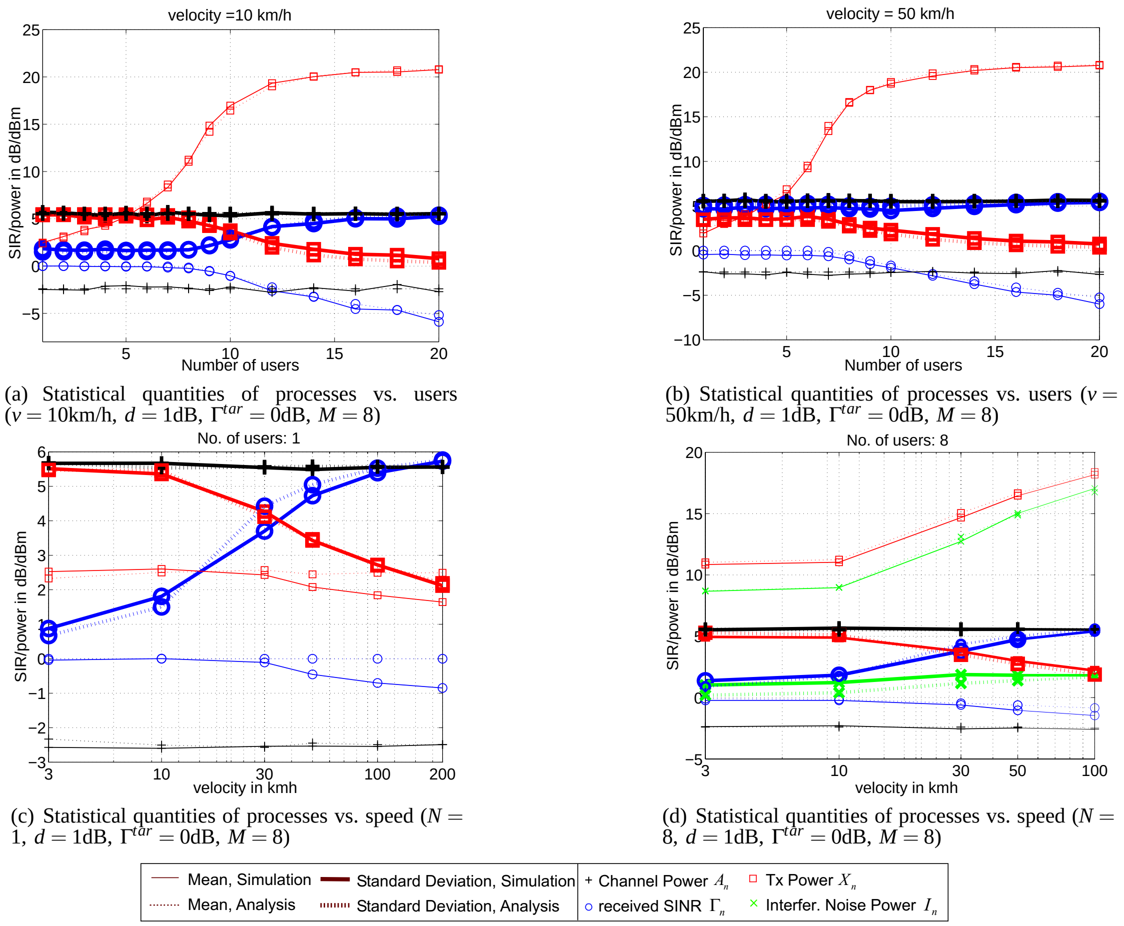
Figures 4(a)-4(d) show some results for the case that all power control loops have the same parameterization, with target SINR's $\Gamma_n^{tar} = \Gamma^{tar} = 0$dB, spreading gains $M_n = M = 8$ and all users moving at the same velocity. The power control stepsize has been chosen to 1dB which corresponds to the 3GPP standard [4]. As an example, Fig. 4(a) shows the behavior of the main statistical system parameters vs. the number of users, resulting from simulation and from analysis, which are obviously in excellent agreement. Already here the reaction of a system exposed to an infeasible load situation can be evaluated, as for $N > 8$ the respective target SINRs can not be reached.

Fig.4(c) shows the behavior vs. the velocity for only one user. In this case there are no errors due to the FW-approximation since it is not incorporated. Obviously, in case of high velocities the mean of the target SINR drops below the given target, although the system is not overloaded. This effect can be explained as follows. In the case of high velocities the transmit power can not be adapted to the inverse of the channel weight process due to the fixed power control stepsize of 1dB and the inherent power control delay. Thus the process R and consequently the process Y are no longer Gaussian. In the limit of high speeds Y becomes close to the process A with respect to its statistical properties. One important characteristic of the process A is, that its pdf is asymmetric in the logarithmical domain, i.e. that the mean and the median are no longer equal. Due to this effect, it is possible, that the output of the slicer Z has zero mean, the system is not driven into the limit, although the mean at the input of the slicer has a mean unequal to zero. This can not be accounted for in our analytical model, since for the linearization of slicer and limiter and for the application of the FW-approximation the signals have been assumed to be Gaussian distributed and thus having a symmetric pdf. Due to this effect all mean values have an offset corresponding to the observed difference in μ_y between simulation and analysis. This deviation between reached SINR and target SINR can be compensated for by the outer power control loop which increases the target SINR properly according to this deviation. This shows that this deviation between target SINR and reached SINR is not caused by the limited system capacity, in contrast it only results from the power control.

Furthermore Fig. 4(c) shows, that for high velocities the variance of the control error approaches the variations of the channel process A_n, which shows the correctness of our model. At low speeds the control error variance approaches 0.5dB due to the stepsize of 1dB.

Fig. 4(d) shows the behavior vs. speed in case of 8 users. In this figure also our analytical model shows an effect of a decreasing mean SINR for increasing velocities. This deviation shown here arises due to the limited system capacity and the increasing interference noise power in case of increasing velocities and cannot be compensated by the outer loop power control. It is superimposed by the effect of the deviation of the reached SINR from the target SINR in case of high velocities due to the asymmetric pdf as it has already been observed at Fig. 4(c). The increasing deviation from the target SINR for increasing velocities due to the limited capacity can also be understood in the following way: With increasing velocities, the mean transmit powers increase, which lead to a decreased system capacity in terms of supportable users. Except for the problem discussed above, the results of our analytical model are in very good accordance with the results obtained by simulations.

In Fig. 4(a) it is obvious that for $N \geq 9$ users the mean received SINR begins to decrease and thus the given target SINR can not be achieved, as it is in accordance with theoretical considerations for static propagation conditions [1]. Fig. 4(b) shows that for a higher velocity ($v = 50$km/h) the achieved SINR already begins to decrease for $N \geq 7$ users. Obviously the capacity in terms of supportable users depends on the velocity and decreases with increasing velocities. This effect can be clearly evaluated with our analytical model. Due to the non ideal power control, the variances of the received signal powers increase with the velocity. Consequently the



(a) Statistical quantities of processes vs. users ($v = 10$km/h, $d = 1$dB, $\Gamma^{tar} = 0$dB, $M = 8$)

(b) Statistical quantities of processes vs. users ($v = 50$km/h, $d = 1$dB, $\Gamma^{tar} = 0$dB, $M = 8$)

(c) Statistical quantities of processes vs. speed ($N = 1$, $d = 1$dB, $\Gamma^{tar} = 0$dB, $M = 8$)

(d) Statistical quantities of processes vs. speed ($N = 8$, $d = 1$dB, $\Gamma^{tar} = 0$dB, $M = 8$)

Fig. 4. Analytical Results (Note: Sometimes analytical results overlap with simulation results)

mean of the interference noise power increases with the velocity, and thus the mean transmit power increases too, see Fig. 4(d). Hence fewer users can be supported since the system is interference limited.

The validity of our analytical model has also been proven for power control stepsizes different from 1dB. Furthermore our model enables the analysis in the case of users moving at different velocities, having different target SINRs, and having different spreading gains. Also in this case, the analytical model delivers accurate results, with the same factor $\alpha = 0.5$ in the FW-approximation.

VI. Conclusion

In this paper an analytical model for a CDMA reverse link with an up/down power control algorithm has been developed. The effect of the mobile speed and the number of users on the mean and variance of the control error and the transmit power has been studied. Furthermore, our model is general as it accounts for users being parameterized with different target SINR's, different spreading gains and different power control stepsizes. Our model incorporates the coupling between the different power control loops due to the interference. The model includes also the limitation of transmit powers.

The results achieved by our analytical model are in excellent agreement with simulation results, and thus the analytical system model enables fast evaluation of the feasibility of a specific scenario and the reaction of the system exposed to this situation. The understanding of this reaction is necessary for the development of better algorithms e.g. to detect infeasible system load situations.

Furthermore the analysis shows that the number of supportable users in a system in a dynamic channel environment using the up/down power control algorithm is less then the theoretical achievable number of users in case of perfect power control. Up to now, SINR measurement is assumed to be perfect, but SINR estimation errors can be introduced to the model as a further input process and thus the model will enable us to study the impact of of different SINR estimators on the system behavior. Furthermore it will be interesting to study the effect on power control if multipath fading is considered. It is expected that due to the RAKE receiver the impact of deep fades will be decreased.

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