Biological Hydrodynamics

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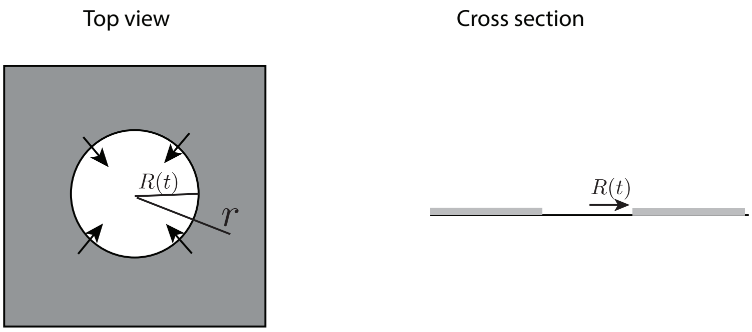
Next Tutorial: Thursday **19th December**, 14:50 - 16:20, MPI PKS Seminar Room 3

Tutorial 7: Wetting and dewetting



In this tutorial, we consider the process of wetting of a thin film on a surface, and the analogy to the opening of a hole by laser ablation in the cytoskeleton at the surface of the cell.

In a wetting process, a thin layer of fluid spreads onto a surface because of the adhesion energy of the fluid to the surface (hence the term wetting). The adhesion energy creates a negative isotropic stress $\sigma^a_{ij} = -W\delta_{ij}$ in the layer ($W > 0$). Here, we approximate the thin layer as a two-dimensional fluid, and stresses refer to two dimensional stresses in the film. We consider a situation where a circular hole is made in an infinite thin film, and surface adhesion drives expansion of the fluid to close the hole. The radius of the hole changes over time and is given by the function $R(t)$. There is a symmetry of rotation such that in polar coordinates (r, θ), all quantities depend on r only. The fluid is not incompressible here.

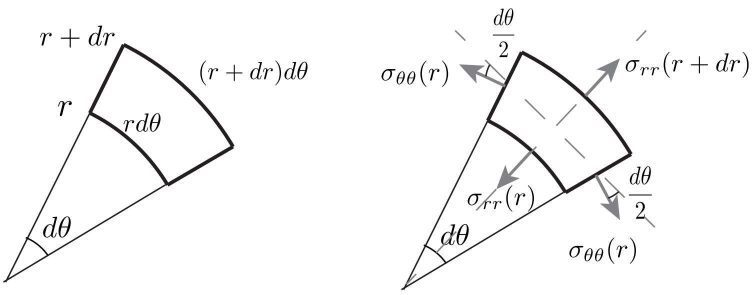
1. We start by deriving the 2D hydrodynamics equation for a compressible fluid. The constitutive equation for the deviatoric stress tensor for a Newton's fluid reads $\sigma^d_{ij} = 2\eta v^d_{ij}$, where η is the shear viscosity, $\sigma^d_{ij} = \sigma_{ij} - \frac{1}{2}\sigma_{kk}\delta_{ij}$ is the deviatoric stress tensor, and $v^d_{ij} = v_{ij} - \frac{1}{2}v_{kk}\delta_{ij}$ is the deviatoric part of the gradient of flow. Using the same principle, write down the equation a phenomenological constitutive equation for the trace of the stress tensor σ_{kk}, by introducing the bulk viscosity η_b.

2. By writing that the total stress is the sum of the deviatoric viscous stress, isotropic viscous stress, and adhesion energy, write down the expression of the total stress.

3. By writing that the sum of forces on a element of fluid contained between r and $r+dr$ and $\theta + d\theta$ has to cancel to satisfy the law of Newton, show that the expression of force balance for the components of the stress in polar coordinates, σ_{rr} and $\sigma_{\theta\theta}$ ($\sigma_{r\theta} = 0$ with rotational symmetry.), reads

$$\partial_r \sigma_{rr} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0 \tag{1}$$

Hint: use the figure below and project the total force on a radial line.



4. The gradient of flow reads in polar coordinates and with rotational symmetry $v_{rr} = \partial_r v_r$, $v_{\theta\theta} = \frac{v_r}{r}$ and $v_{r\theta} = 0$. From these expressions and the force balance equation obtained in the previous question, derive a differential equation for the flow.

5. Solve for the flow profile by using the boundary conditions at infinity ($v_r(r) \to 0$ when $r \to \infty$) and at the edge of the hole ($\sigma_{rr}(R) = 0$)). Hint: Show that the flow obeys a Cauchy-Euler equation, and use the appropriate ansatz to solve it.

6. By using the equation for the motion of the boundary, $v_r(R(t)) = \frac{dR}{dt}(t)$, write down and solve a differential equation for $R(t)$. Which parameters set the characteristic time scale of contraction?

7. In the cytoskeleton at the cell surface, molecular motors create a positive stress, such that the previous description still applies with $W < 0$. What happens when a circular hole is made in the cytoskeleton?